THE UNIVERSITY OF TOKYO

東京大学 大学院
理学系研究科・理学部
SCHOOL OF SCIENCE, THE UNIVERSITY OF TOKYO

東京大学大学院 物理学専攻 集中講義
17-19, May 2011

グラフェンの物理

物理学専攻
青木 秀夫

TOC

- 一体問題：

歴史的背景（グラファイト、グラフェン）
有効質量近似とDirac分散
　Fermion doublingとNielsen-Ninomiya theorem
　Tight-binding modelに依らない定式化
カイラル対称性
磁場中のグラフェン
　ランダウ準位、index theorem
グラフェン量子ホール効果
グラフェンの実験
　試料、STM, ARPES, 光学物性、…
Ripple
様々なDirac系
　3次元系（GIC, MgB_2）との比較
　1次元系（carbon nanotube）との比較
　電子・正孔非対称なDirac cone
　傾いたconeと、一般化されたカイラル対称性
　2層グラフェン
新奇物性の提案
　Terahertz QHE
　円偏光下のゼロ磁場Hall効果

- 多体問題：
　分数量子ホール状態、Chiral condensate、・・・

Bibliography

***Historical**

1) P.R. Wallace, Phys. Rev. **71**, 622 (1947).
2) W.H. Lomer, Proc. Roy. Soc. (London) **330**, A227 (1955).
3) J.W. McClure, Phys. Rev. **104**, 666 (1956).
4) J.C. Slonczewsky and P.R. Weiss, Phys. Rev. **109**, 272 (1958).

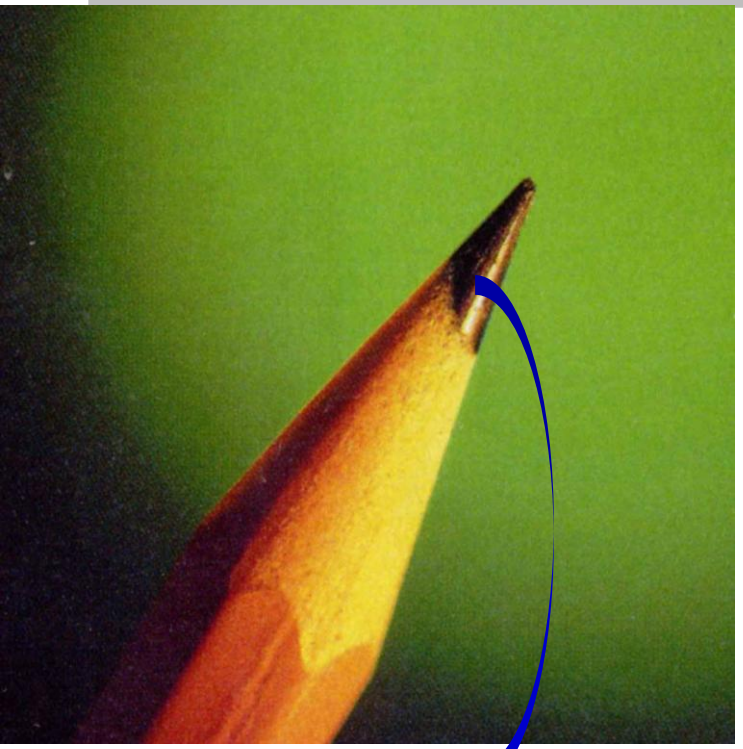
***Recent**

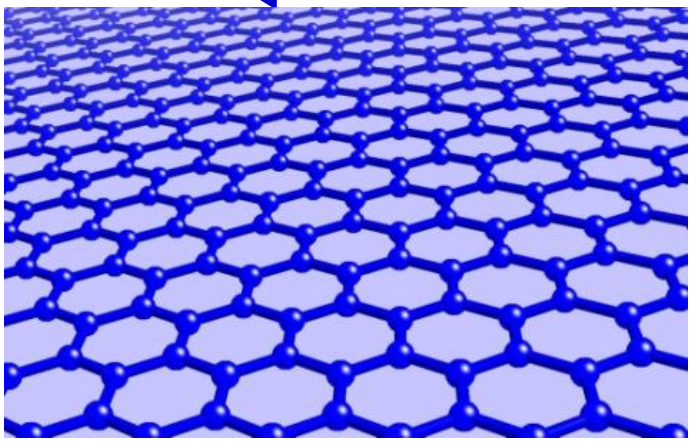
5) K.S. Novoselov et al, Nature **438**, 197 (2005).
6) Y. Zhang, Y.-W. Tan, H.L. Stormer and P. Kim, Nature **438**, 197 (2005).
7) A.K. Geim and K.S. Novoselov, Nature Materials **6**, 183 (2007).
8) A.H. Castro Neto et al, Rev. Mod. Phys. **81**, 109 (2009).
9) D.S.L. Abergel, V. Apalkov, J. Berashevich, K. Ziegler and T. Chakraborty, Adv. Phys. **59**, 261 (2010).
10) Y. Hatsugai, T. Fukui and H. Aoki, Phys. Rev. B **74**, 205414 (2006).
11) H. Aoki and M. Dresselhaus (editors): *Physics of Graphene* (Springer, to be published).
12) 斉木幸一朗、徳本洋志(編)「グラフェンの機能と応用展望」(CMC出版、2009)。
13) 初貝安弘、青木秀夫:グラフェンの物理、固体物理 **45**, 457 (2010)。
14) 青木秀夫:ディラック電子、固体物理 **45**, 753 (2010)。
15)岡 隆史、青木秀夫:グラフェンのトポロジカルな性質とその光制御、表面科学 **32**, 196 (2011)。

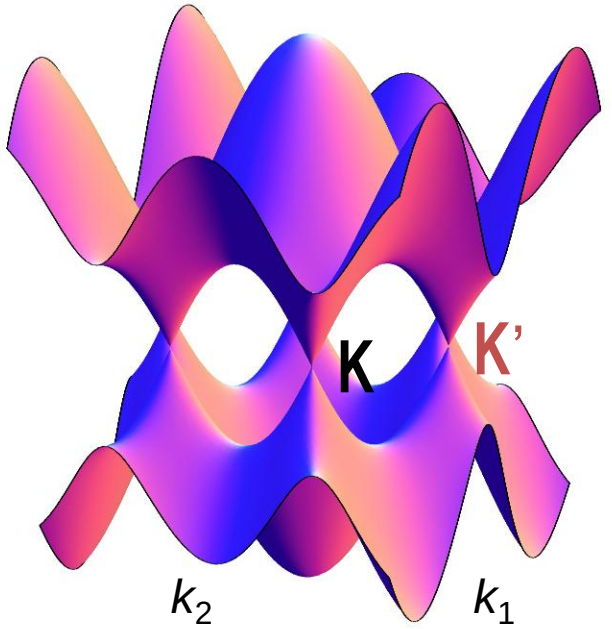
*** Quantum Hall effect**

16) 中島龍也、青木秀夫:「分数量子ホール効果」(東京大学出版会、1999)。
17) Hideo Aoki in *Comprehensive Semiconductor Science & Technology* (Elsevier, 2011).

Graphene – massless Dirac particles







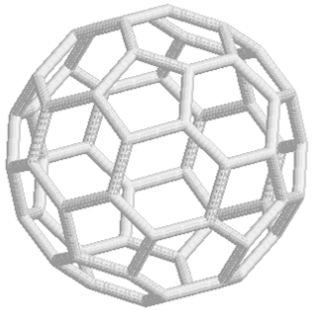
質量がゼロのDirac粒子
～ Weyl neutrino



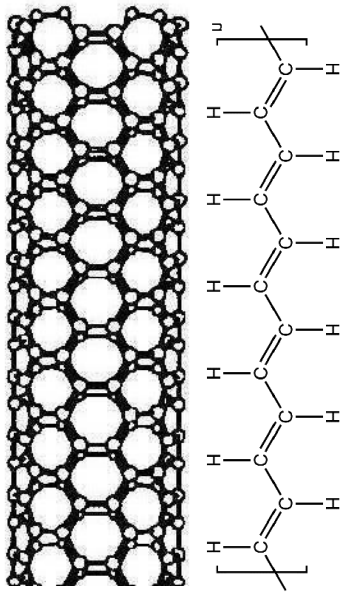
青木 秀夫: グラフェンの物理

基本的なこと

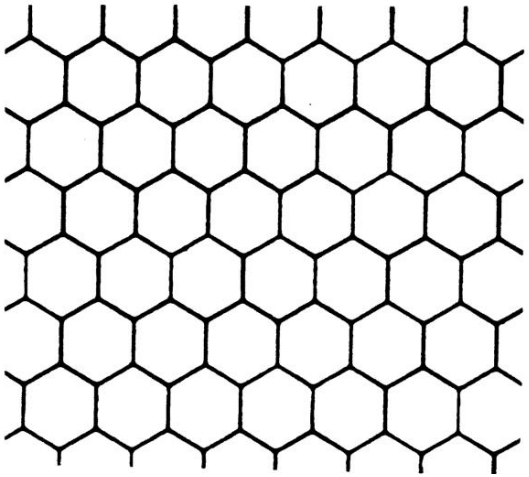
炭素の様々な形態



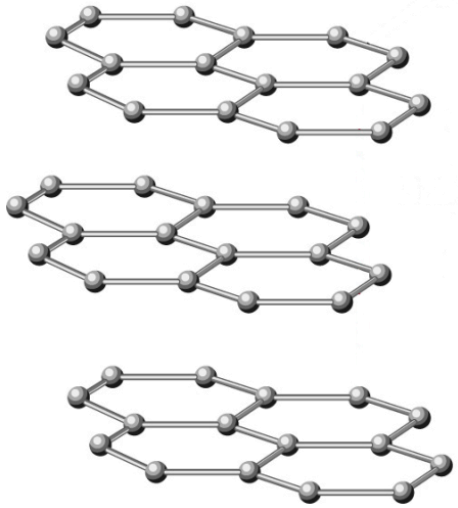
(a) Fullerene (0D)



(b) Nanotube (1D)
Polyacetylene



(c) Graphene (2D)

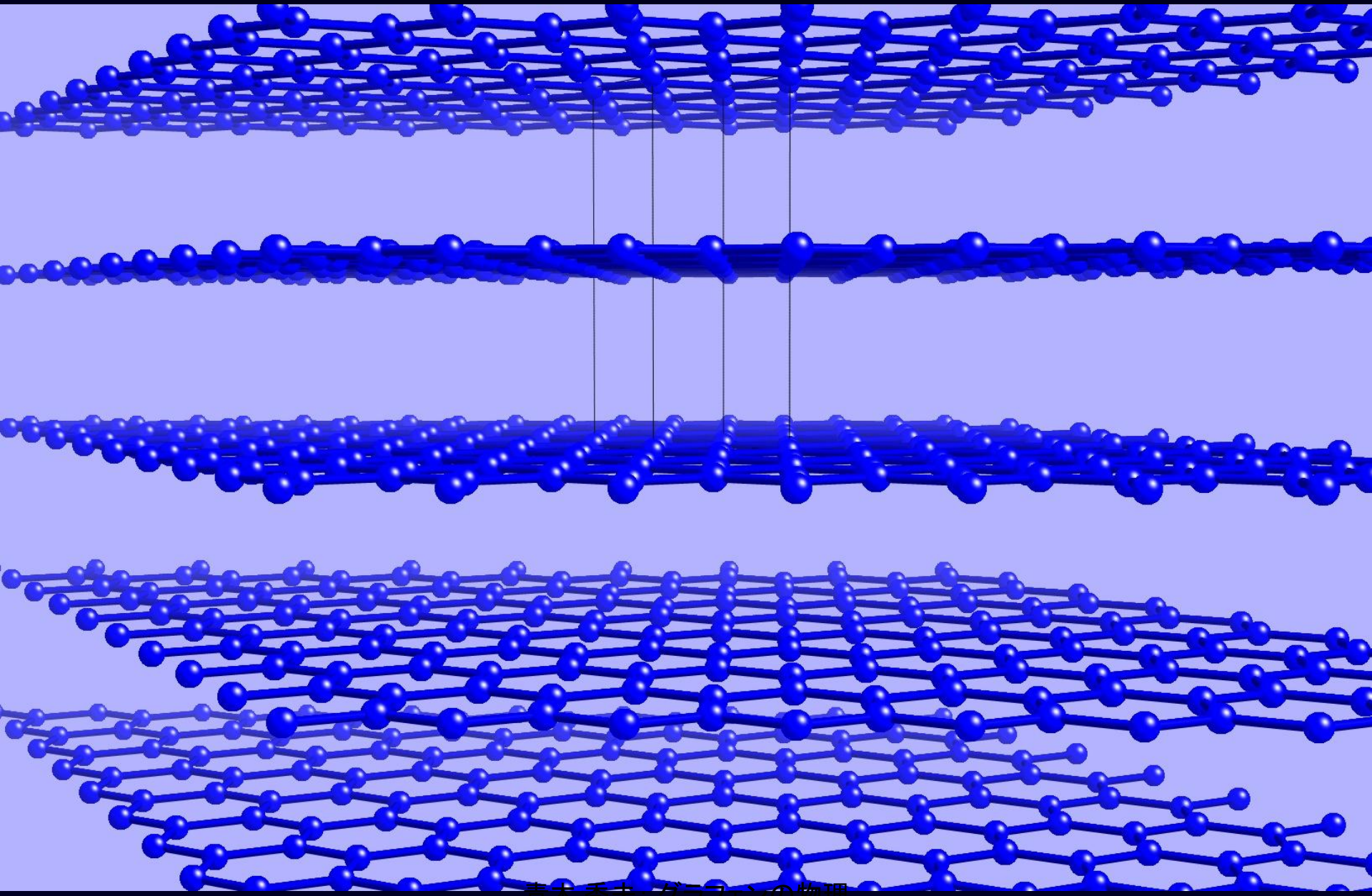


(d) Graphite (3D)

Graphite

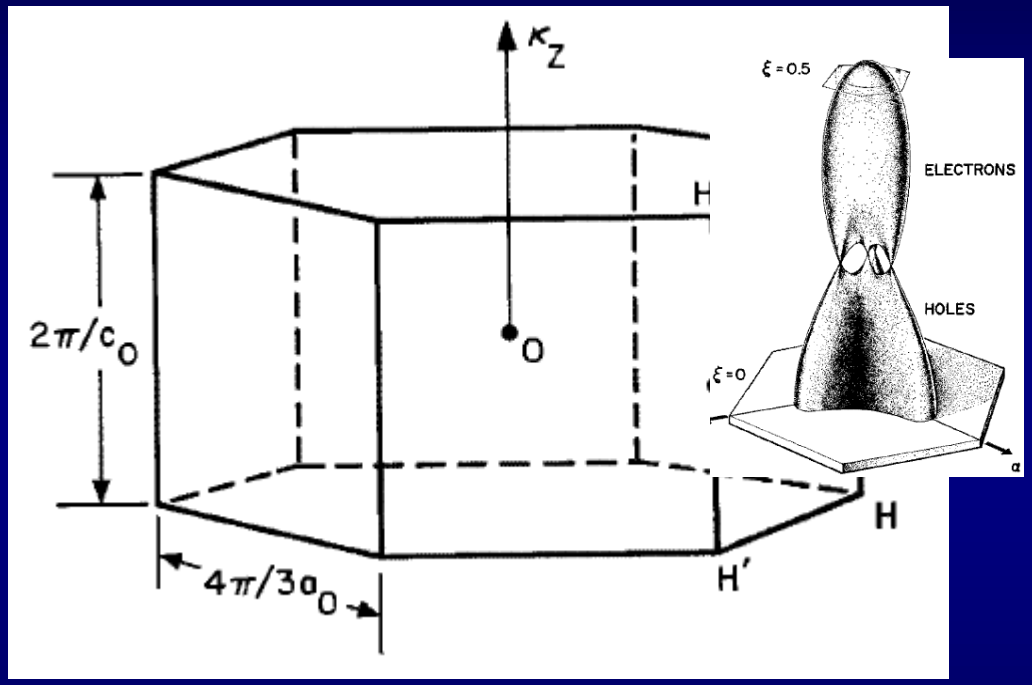


Wada collection (b. 1856) © Univ Tokyo Museum



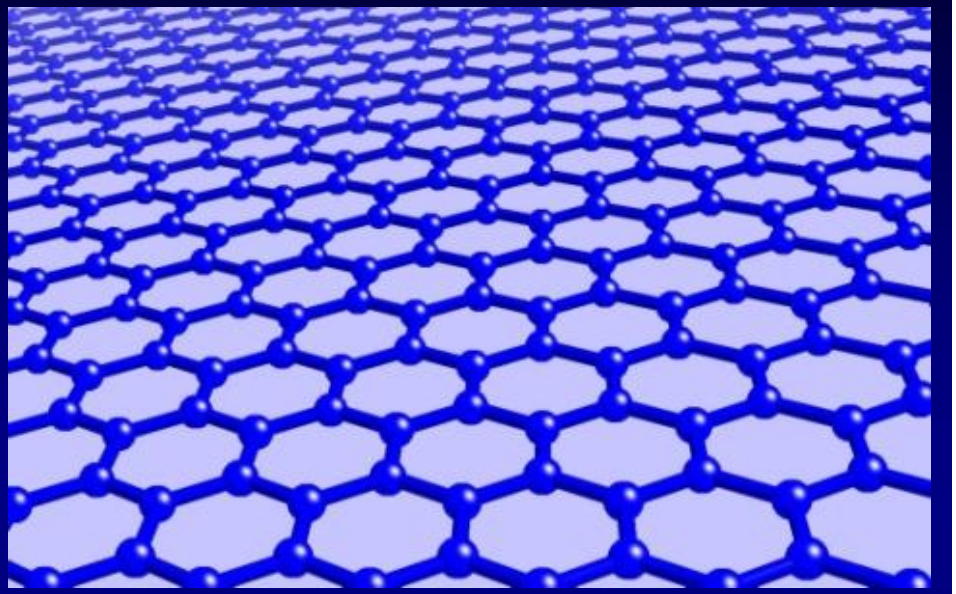
青木 秀夫：グラフェンの物理

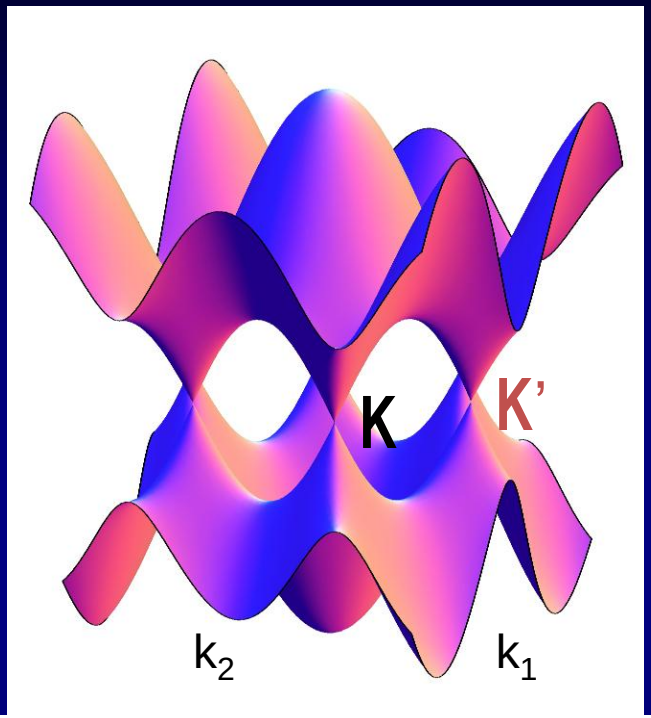
Graphiteのフェルミ面



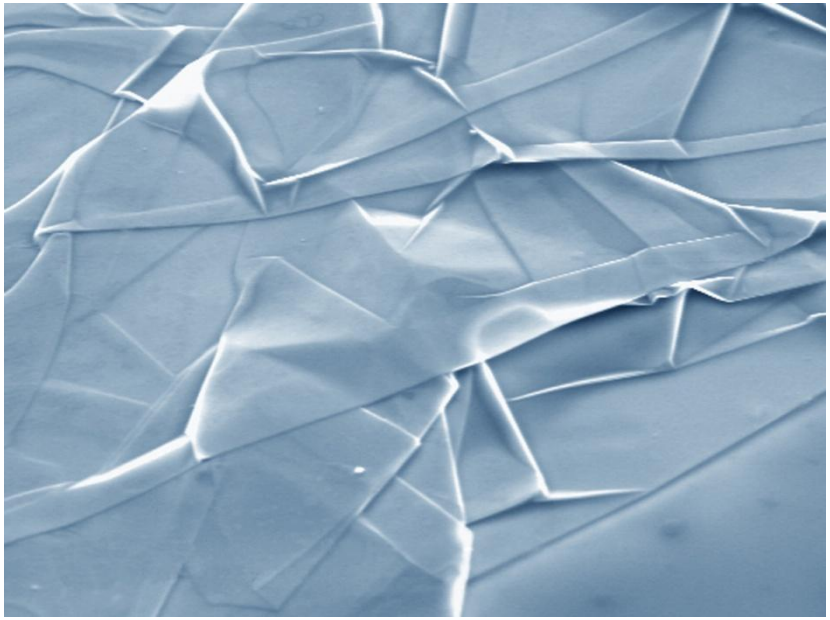
Dresselhaus & Mavroides, 1964

Graphene band structure

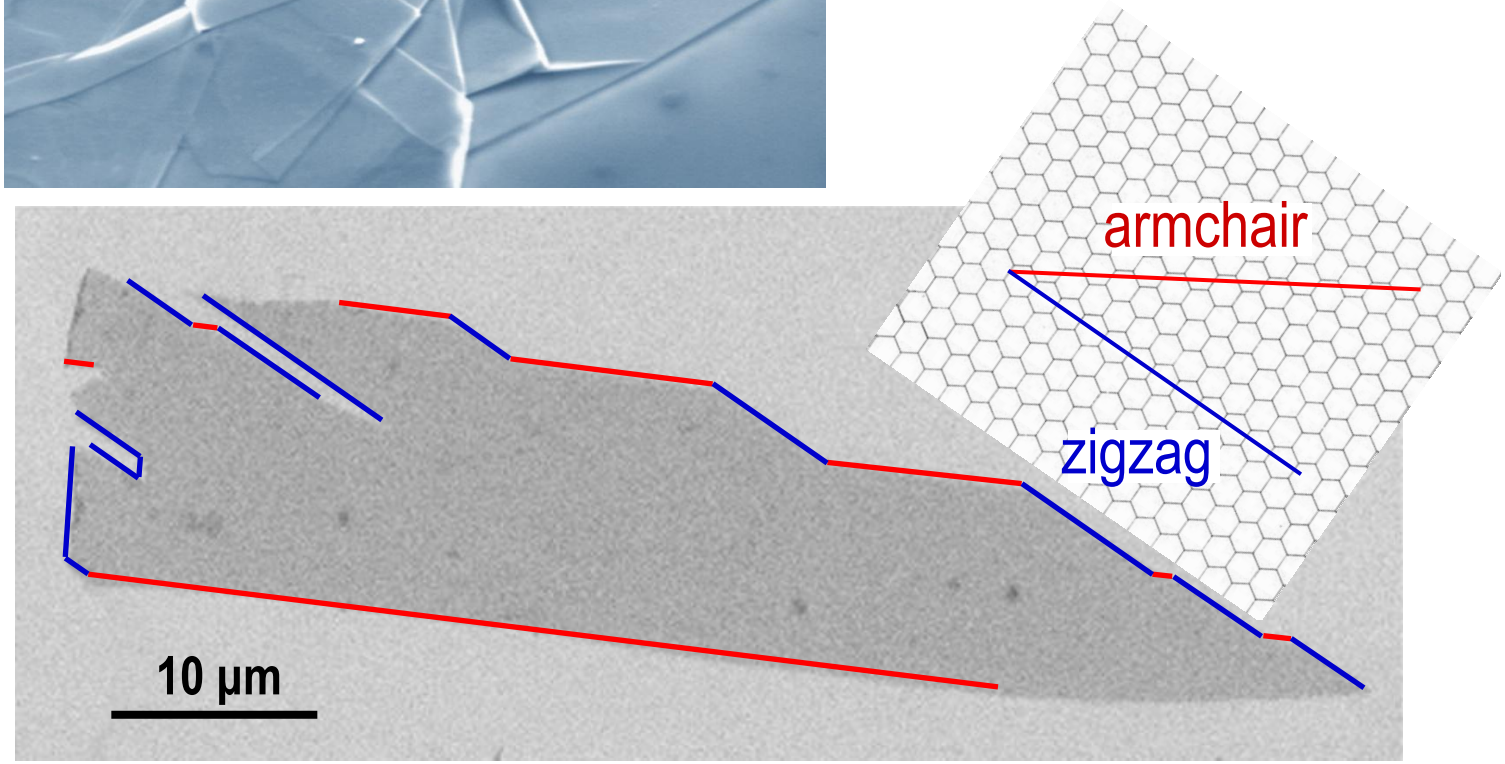




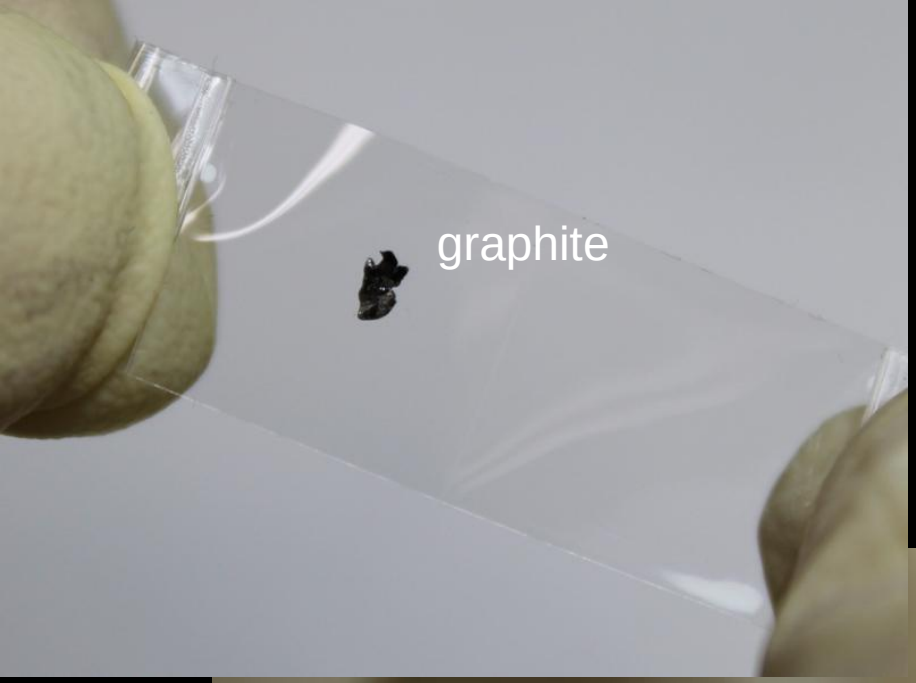
$$E_{g2D}(k_x, k_y) = \pm t \left\{ 1 + 4\cos\left(\frac{\sqrt{3}k_x a}{2}\right)\cos\left(\frac{k_y a}{2}\right) + 4\cos^2\left(\frac{k_y a}{2}\right) \right\}^{1/2}$$



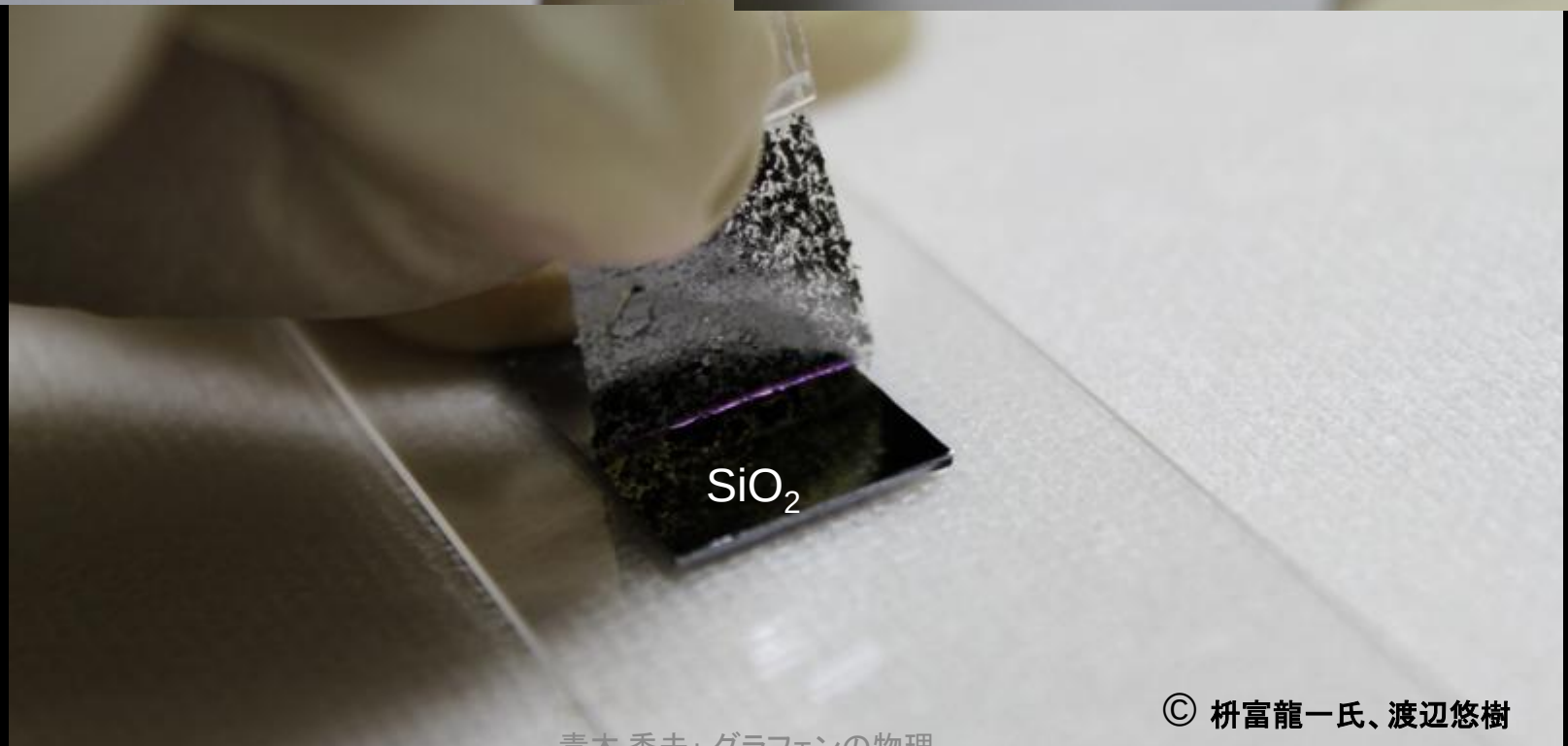
(Novoselov et al, Science 2006, suppl info))



A graphene sample
(Geim & Novoselov, Nature Materials 2007)







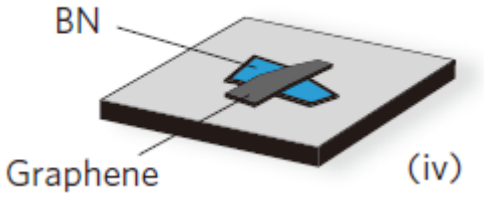
© 枡富龍一氏、渡辺悠樹

More recently:

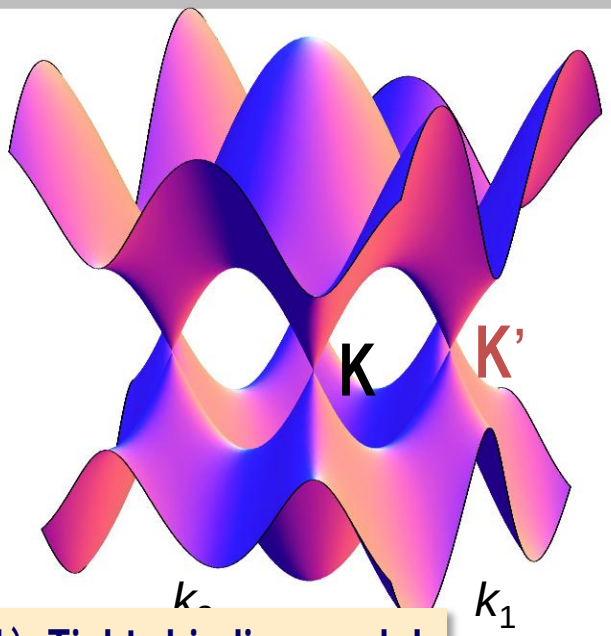
Epitaxial graphene on SiC

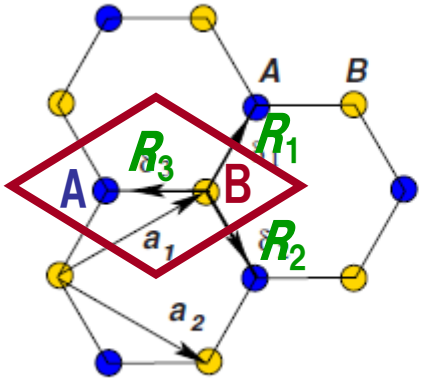
CVD (chemical vapour deposition)

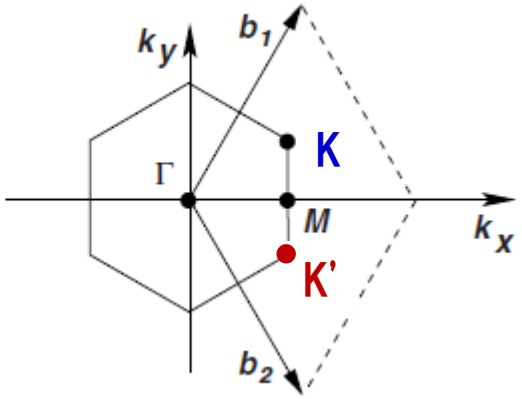
Graphene on BN (either CVD or transfer)



How does the massless Dirac appear on honeycomb







(1) Tight-binding model

$$\mathcal{H}_{AB} = t(e^{i\vec{k}\cdot\vec{R}_1} + e^{i\vec{k}\cdot\vec{R}_2} + e^{i\vec{k}\cdot\vec{R}_3}) \qquad t(\approx 2.8\ \mathrm{eV}) \qquad v_F \approx 10^6\ \mathrm{ms}^{-1}$$

$$H = \begin{pmatrix} 0 & H_{AB} \\ H_{AB} & 0 \end{pmatrix} \qquad E_{g2D}(k_x, k_y) = \pm t \left\{ 1 + 4\cos\left(\frac{\sqrt{3}k_x a}{2}\right)\cos\left(\frac{k_y a}{2}\right) + 4\cos^2\left(\frac{k_y a}{2}\right) \right\}^{1/2}$$

(2) Group theory (Lomer, Proc Roy Soc 1955)

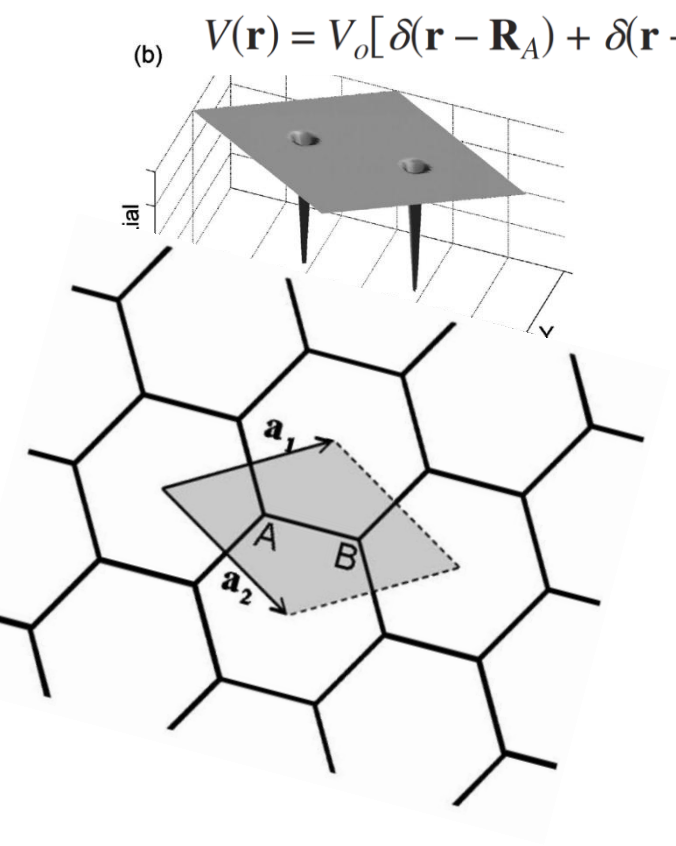
2-dim representation at K and K'

青木 秀夫: グラフェンの物理

Only the honeycomb symmetry matters

We don't have to take the tight-binding model

→ **Honeycomb array of atomic potentials**



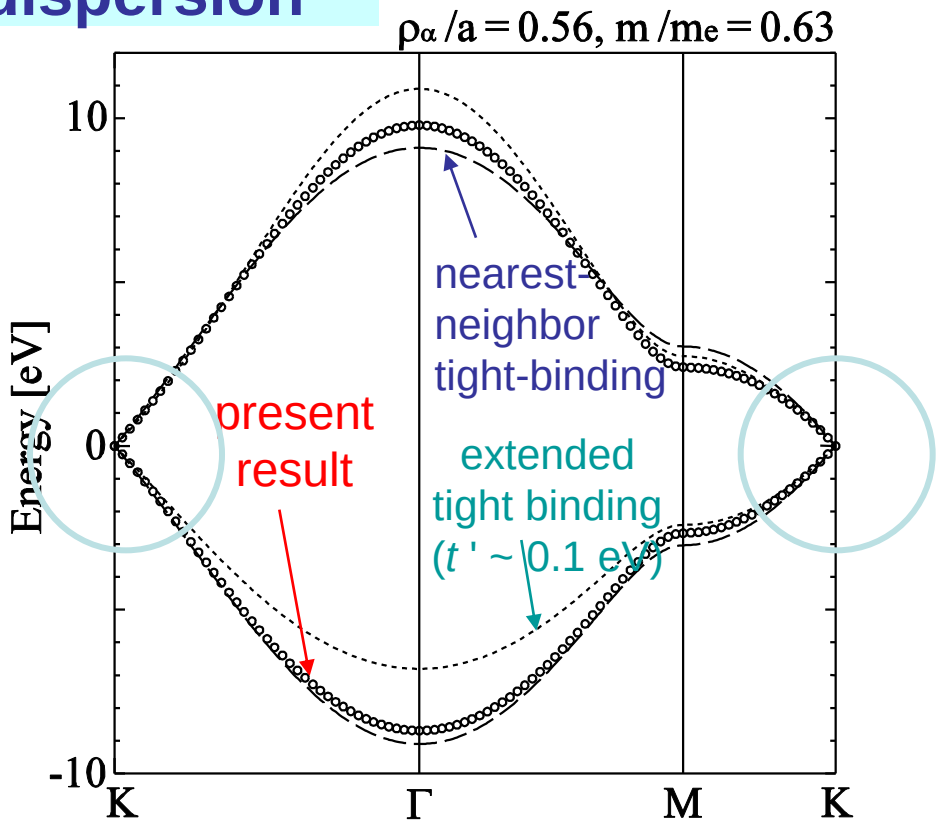
Hsu and Reichl (PRB 2005)

performed a plane-wave expansion with a momentum cutoff
→ zero-gap at K only approximate.

Singular form of $\delta^{2D}(\boldsymbol{r})$ requires a renormalisation of potential strength & a regularisation of its form.

Nakajima & Aoki (Physica E 2008)

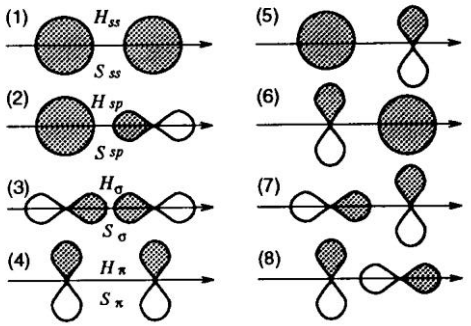
B=0 band dispersion

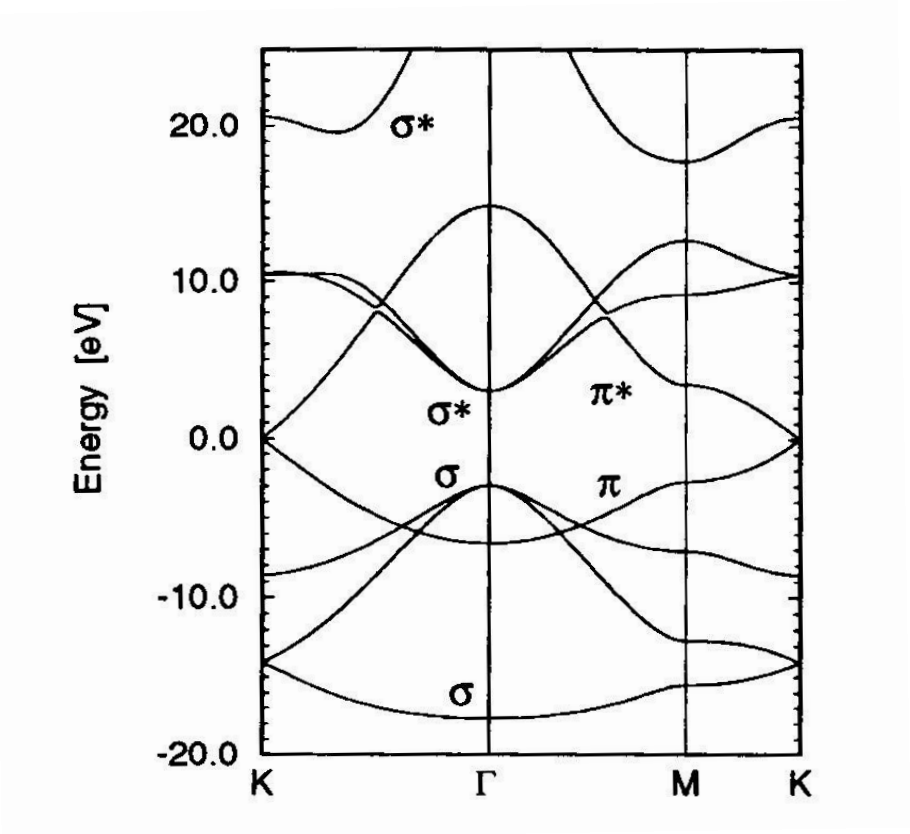


massless Dirac analytically reproduced

Nakajima & Aoki (Physica E 2008)

Full sp^2 calculation

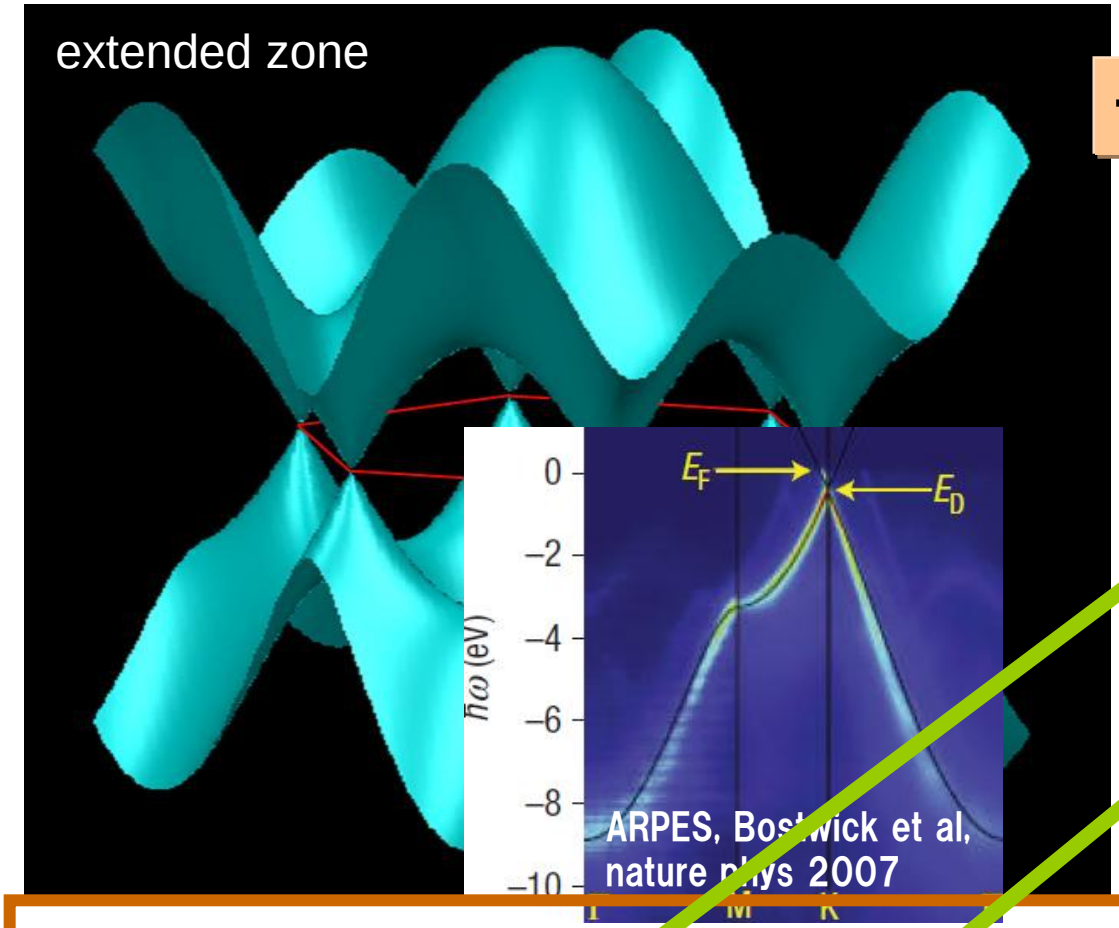




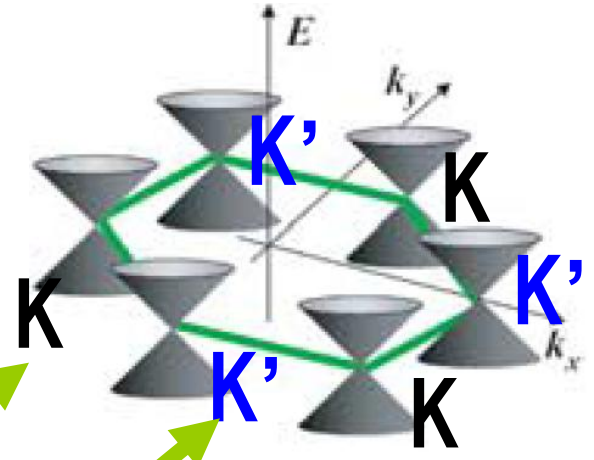
(Saito, Dresselhaus[2]: *Physical Properties of carbon nanotubes*)

青木 秀夫: グラフェンの物理

Dirac particles in graphene



two massless Dirac points



T-reversal: K ⇔ K'

＊ k・p Hamiltonian (effective-mass formalism)
[Wallece, PR 71, 622 (1947)]

$$H_{\mathrm{K}} = v_{\mathrm{F}}(\sigma_x p_x + \sigma_y p_y) = v_{\mathrm{F}} \begin{pmatrix} 0 & p_x - ip_y \\ p_x + ip_y & 0 \end{pmatrix}$$

$$H_{\mathrm{K}'} = v_{\mathrm{F}}(-\sigma_x p_x + \sigma_y p_y) = v_{\mathrm{F}} \begin{pmatrix} 0 & p_x + ip_y \\ p_x - ip_y & 0 \end{pmatrix}$$

青木 秀夫: グラフェンの物理

Massless Dirac eqn for graphene

(Lomer, Proc Roy Soc 1955)

$$\mathcal{H} = \frac{\gamma}{\hbar}\begin{pmatrix} 0 & \pi_x - i\pi_y & 0 & 0 \\ \pi_x + i\pi_y & 0 & 0 & 0 \\ 0 & 0 & 0 & \pi_x + i\pi_y \\ 0 & 0 & \pi_x - i\pi_y & 0 \end{pmatrix}$$

K (rows 1–2), K' (rows 3–4); columns: K, K'

$$\gamma = \sqrt{3}a\ t\ /2 \qquad \hat{\boldsymbol{\pi}} = \hat{\mathbf{p}} + e\mathbf{A}/c \qquad \phi = {}^{t}(F_A^K, F_B^K, F_A^{K'}, F_B^{K'})$$

$$\mathbf{F}_{s\mathbf{k}}^{K}(\mathbf{r}) = \frac{1}{\sqrt{2}L}\exp(i\mathbf{k}\cdot\mathbf{r})\begin{pmatrix} s \\ e^{i\varphi(\mathbf{k})} \\ 0 \\ 0, \end{pmatrix} \qquad \mathbf{F}_{s\mathbf{k}}^{K'}(\mathbf{r}) = \frac{1}{\sqrt{2}L}\exp(i\mathbf{k}\cdot\mathbf{r})\begin{pmatrix} 0 \\ 0 \\ e^{i\varphi(\mathbf{k})} \\ s \end{pmatrix}$$

+ *B* [McClure, PR 104, 666 (1956)]

$$\mathbf{F}_{nk}^{K}(\mathbf{r}) = \frac{C_n}{\sqrt{L}}\exp(-iky)\begin{pmatrix} \mathrm{sgn}(n)i^{|n|-1}\phi_{|n|-1} \\ i^{|n|}\phi_{|n|} \\ 0 \\ 0 \end{pmatrix} \qquad \mathbf{F}_{nk}^{K'}(\mathbf{r}) = \frac{C_n}{\sqrt{L}}\exp(-iky)\begin{pmatrix} 0 \\ 0 \\ i^{|n|}\phi_{|n|} \\ \mathrm{sgn}(n)i^{|n|-1}\phi_{|n|-1} \end{pmatrix}$$

青木 秀夫: グラフェンの物理

Atiyah-Singer index theorem and the n=0 graphene Landau level

$$\mathrm{K} = \begin{pmatrix} 0 & D^\dagger \\ D & 0 \end{pmatrix} \qquad \gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\mathrm{K}^2 = \begin{pmatrix} D^\dagger D & 0 \\ 0 & DD^\dagger \end{pmatrix}$$

Novoselof et al, Nature 2005;
Pachos et al 2007; Park 2010)

D^+D and DD^+ have the same nonzero eigenvalues, λ_+, λ_-, respectively

$$DD^\dagger\psi = \lambda\psi \longrightarrow D^\dagger D(D^\dagger\psi) = \lambda(D^\dagger\psi)$$

If $D^+\psi$ is already zero, we have to count them separately.
Now, we have, for an arbitrary t,

$$\mathrm{Tr}(\gamma_5 e^{-t\mathrm{K}^2}) = \mathrm{Tr}(e^{-tD^\dagger D}) - \mathrm{Tr}(e^{-tDD^\dagger}) = \sum_{\lambda_+} e^{-t\lambda_+} - \sum_{\lambda_-} e^{-t\lambda_-}$$

For λ ¥neq 0 λ_+ and λ_- cancel with each other, leaving us with

$\mathrm{Tr}(\gamma_5 e^{-t\mathrm{K}^2}) = \eta_+ - \eta_-$ where η_+, η_- are # of zeromodes for D, D^+ respectively

We have in fact to calculate Index(K). If we put

$$D = -ie_\nu^\mu \sigma^\nu (\nabla_\mu - ieA_\mu)$$

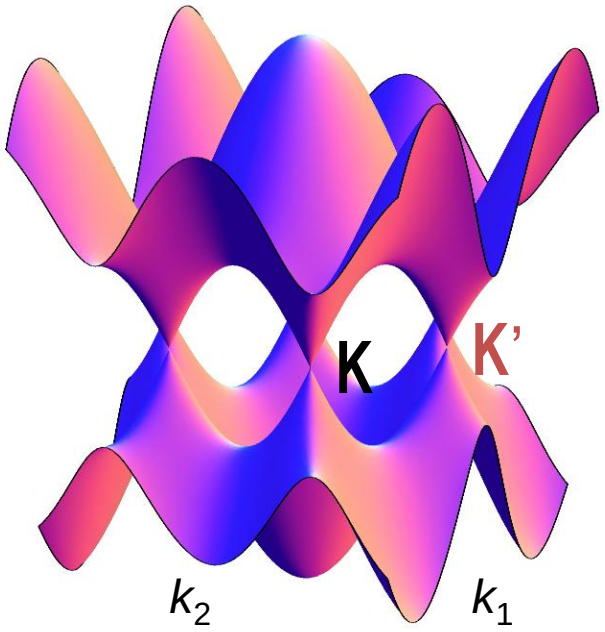
we can show that

$$\mathrm{Index}(\mathrm{K}) = \nu_+ - \nu_- = \frac{1}{2\pi}\int\int \mathrm{B}\cdot dS$$

Chirality in graphene

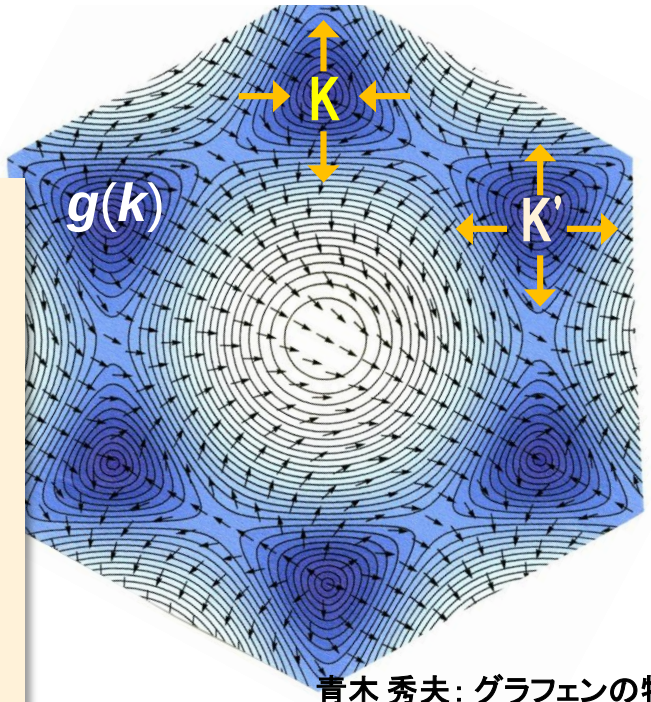
$$\mathcal{H} = \sum_{\mu=1}^{d'} \mathbf{c}_{\boldsymbol{k}}^{+} g_{\mu}(\boldsymbol{k}) \sigma_{\mu} \mathbf{c}_{\boldsymbol{k}}$$

→ Dirac points at $\boldsymbol{g}(\boldsymbol{k})=0$



d' = spatial dim d → Poincare-Hopf th

Σ sgn (det($\partial\, g_\mu / \partial\, k_\nu$)) = 0
→ vortices and antiv appear in pairs
(ie, Nielsen-Ninomiya th)



Chiral symmetry

$\{H,\Gamma\}=0 \quad \Gamma = \sigma_z$

$\Rightarrow \quad E:|\psi\rangle \quad -E:\Gamma|\psi\rangle$

E=0 is special

can be made eigenstates of Γ:

$\gamma\psi_{\pm} = \pm\psi_{\pm}, \quad \psi_{\pm} = (1 \pm \gamma)\psi_{E=0}$

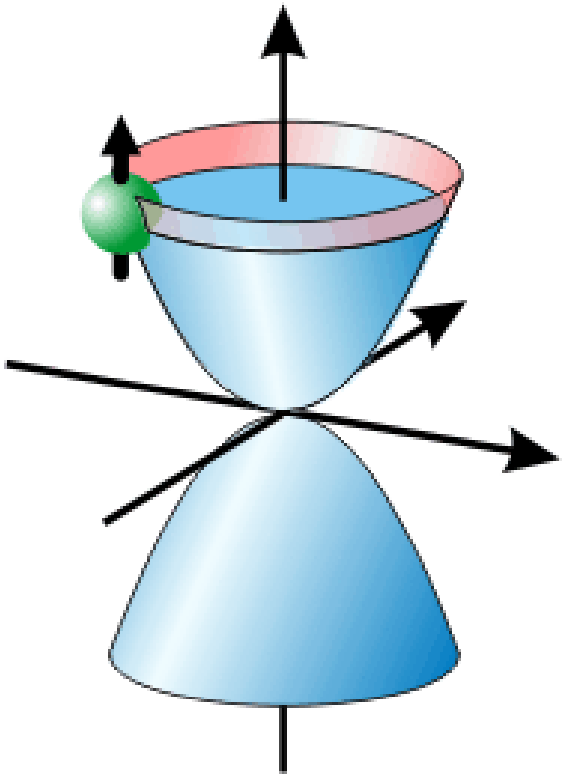
(3+1)	(2+1)
γ_0	σ_x
γ_1	σ_y
γ_2	
γ_3	
γ_4	
γ_5	σ_z

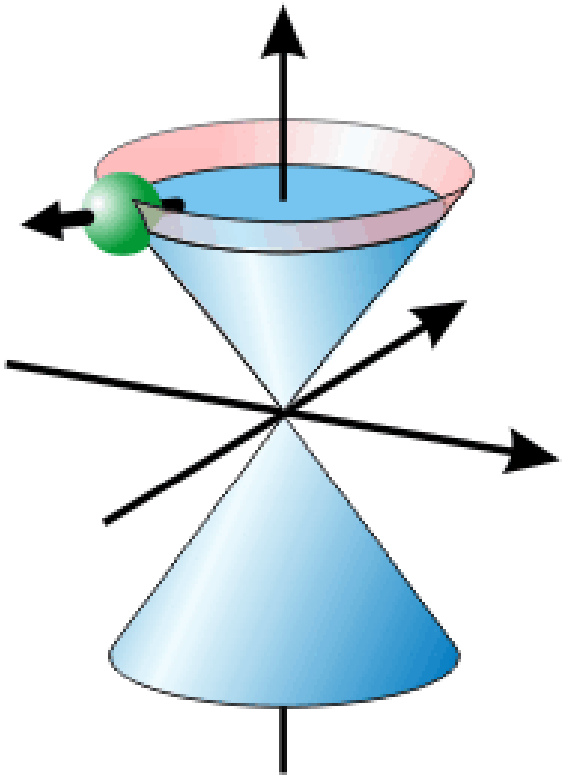
青木 秀夫: グラフェンの物理

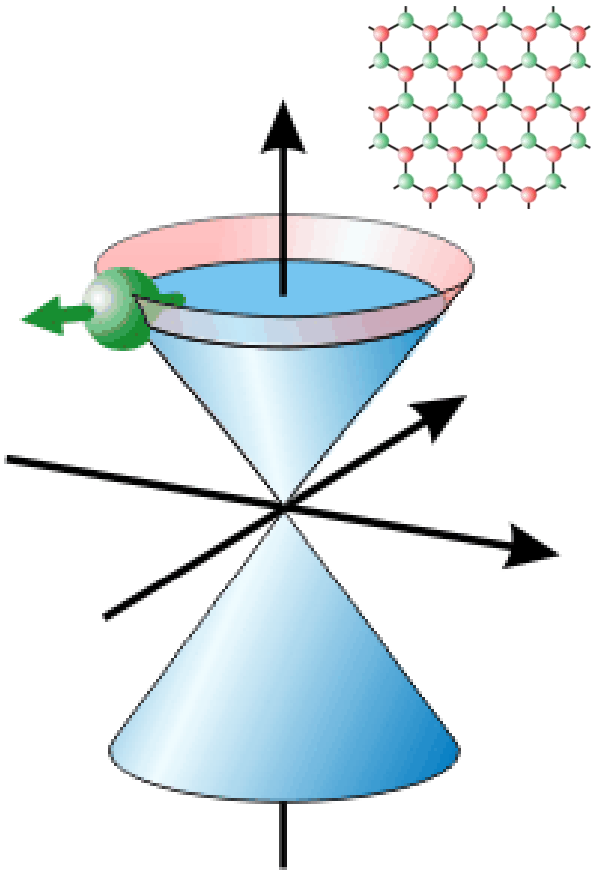
"Schrödinger fermions"

real Dirac fermions

Dirac fermions in graphene







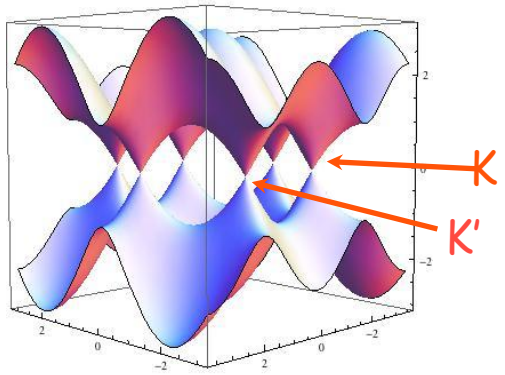
(c) Andre Geim

Chiral symmetry (also applicable to disordered systems)

$$\{\gamma, H\} = 0 \qquad \gamma^2 = 1$$

$$H\gamma\psi = -E\gamma\psi \text{ if } H\psi = E\psi$$

2D Honeycomb Lattice

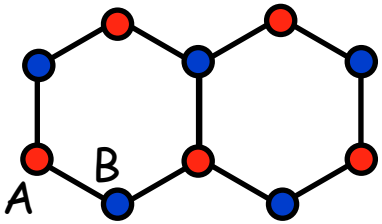


➡ **Zero Energy state : an eigenstate of γ**

$\gamma\psi_{E=0} = \pm\psi_{E=0}$ Special at E=0

Exact Particle-Hole Symmetry in Energy Spectrum

Ex. Honeycomb lattice (graphene)



Bipartite structure (A - B sub-lattice)

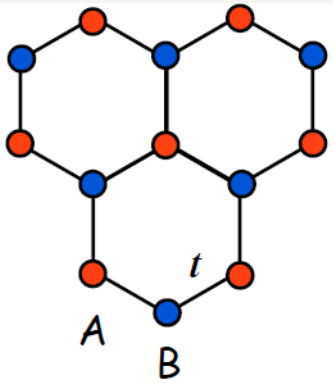
$$H = -\sum_{<i,j>} t_{ij} c_i^{\dagger} c_j$$

Chiral symmetry for honeycomb lattice

Chiral operator

$$\gamma = \exp(i\pi \sum_{i \in A} c_i^\dagger c_i)$$

$$\Gamma c_i \Gamma^{-1} = c_i \quad i \in A \text{ sublattice}$$
$$\Gamma c_i \Gamma^{-1} = -c_i \quad i \in B \text{ sublattice}$$
$$\Gamma^2 = 1$$



Chiral symmetry

$$\{H, \Gamma\} = 0$$

Pairs of eigenstates

$$\Gamma H \Gamma^{-1} = -H \Longrightarrow E : |\psi\rangle \Longleftrightarrow -E : \Gamma|\psi\rangle$$

E=0 is special

$|\psi\rangle, \Gamma|\psi\rangle$: degenerate states

can be made eigenstates of Γ: $\Gamma|\psi_\pm\rangle = \pm|\psi_\pm\rangle$ $|\psi_\pm\rangle = |\psi\rangle \pm \Gamma|\psi\rangle$

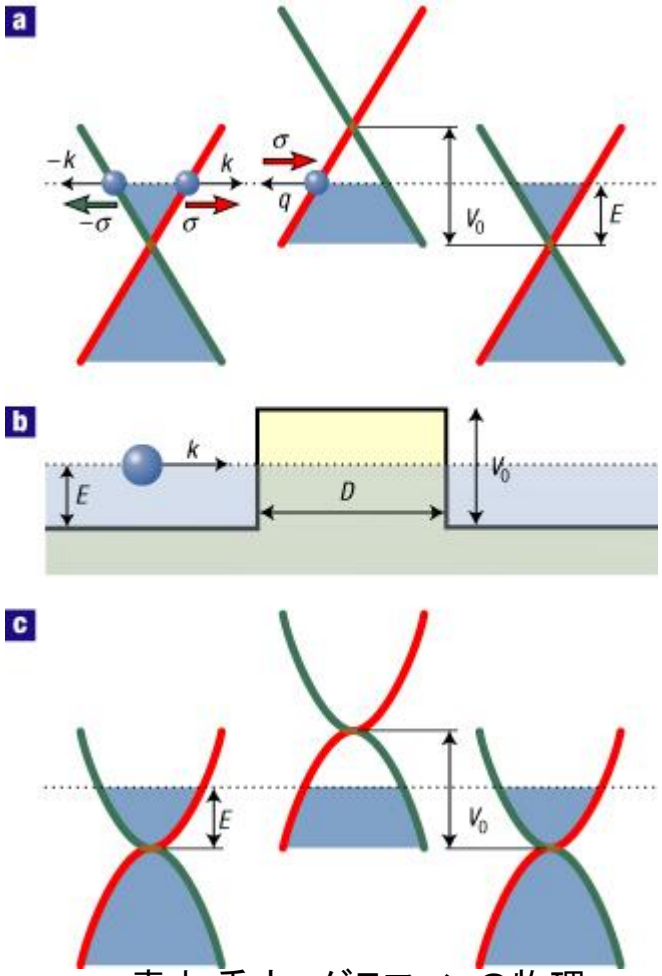
ψ_+ (ψ_-) has its amplitude only on B (A) sub-lattice

Effective (Dirac) field theory

$$\Gamma = \sigma_z$$

Klein's paradox for massless Dirac particles

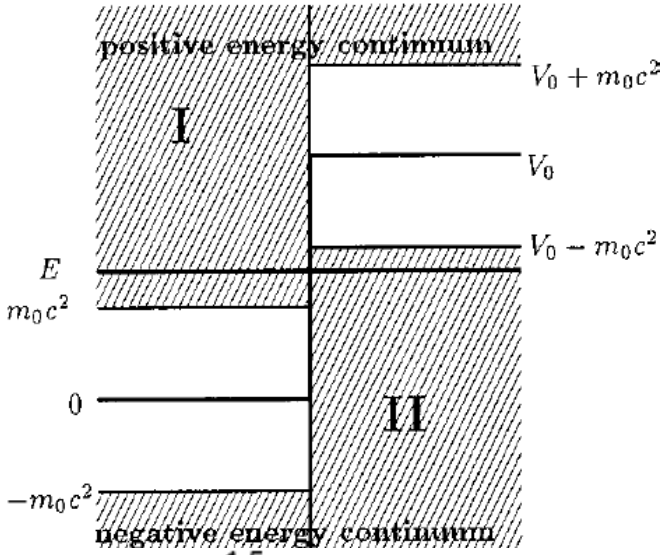
(Katsnelson et al, Nature phys 2006;
see also Young & Kim, Nature phys 2009)



青木 秀夫: グラフェンの物理

Klein's paradox for massive Dirac particles

(Nitta et al, Am J Phys 1999)



+ B [McClure, PR 104, 666 (1956)]

$$\mathbf{F}_{nk}^{K}(\mathbf{r}) = \frac{C_n}{\sqrt{L}} \exp(-iky) \begin{pmatrix} \mathrm{sgn}(n) i^{|n|-1} \phi_{|n|-1} \\ i^{|n|} \phi_{|n|} \\ 0 \\ 0 \end{pmatrix} \qquad \mathbf{F}_{nk}^{K'}(\mathbf{r}) = \frac{C_n}{\sqrt{L}} \exp(-iky) \begin{pmatrix} 0 \\ 0 \\ i^{|n|} \phi_{|n|} \\ \mathrm{sgn}(n) i^{|n|-1} \phi_{|n|-1} \end{pmatrix}$$

$$\phi_{n,k}(x) = (2^n n! \sqrt{\pi} l)^{-1/2} e^{-z^2/2} H_n(z), \quad \text{with} \quad z = (x + kl^2)/l$$

Hermite polynomial

$$C_n = \begin{cases} 1 & (n = 0) \\ 1/\sqrt{2} & (n \neq 0), \end{cases} \qquad \mathrm{sgn}(n) = \begin{cases} 0 & (n = 0) \\ n/|n| & (n \neq 0). \end{cases}$$

Eigenenergies (Landau levels)

$$\varepsilon_n = \hbar \omega_B \, \mathrm{sgn}(n) \sqrt{|n|},$$

$n = 0, \pm 1, \ldots,$: Landau index

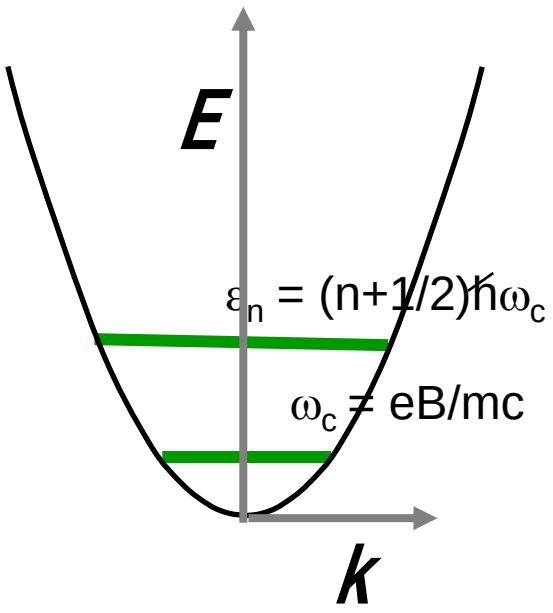
$\hbar \omega_B = \sqrt{2} \gamma / l$ with $l = \sqrt{\hbar / eB}$

cyclotron energy　　magnetic length

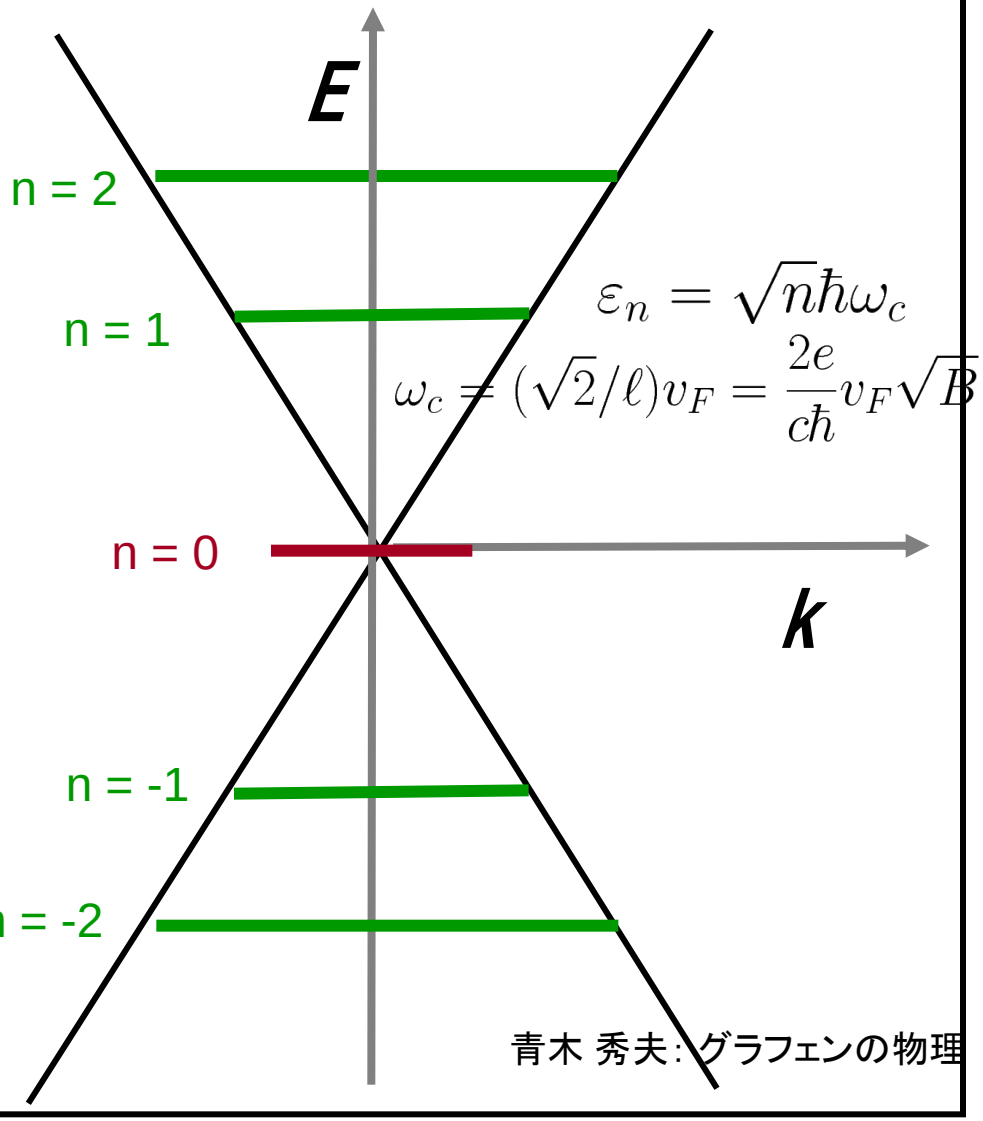
青木 秀夫: グラフェンの物理

Landau levels

通常の量子ホール系



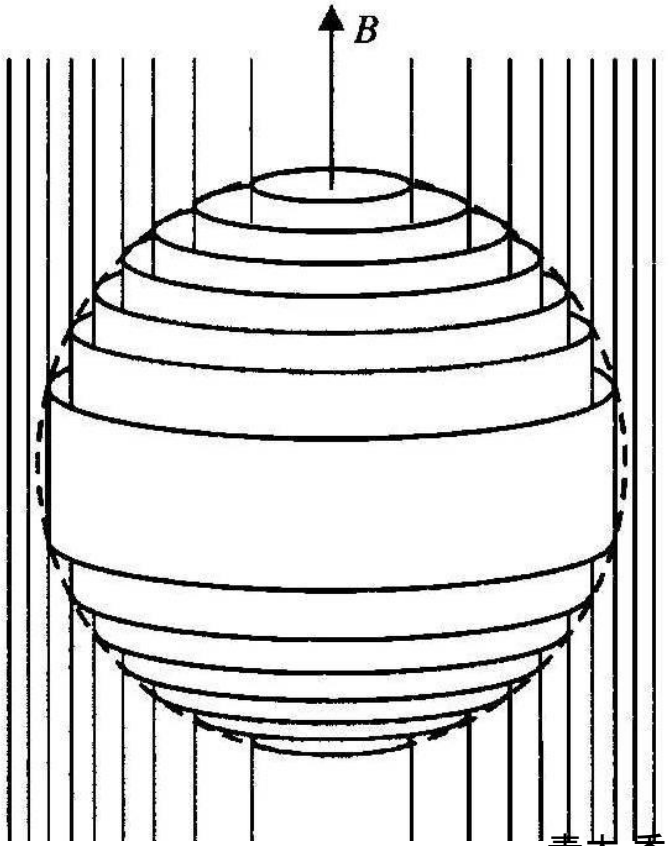
磁場中グラフェン



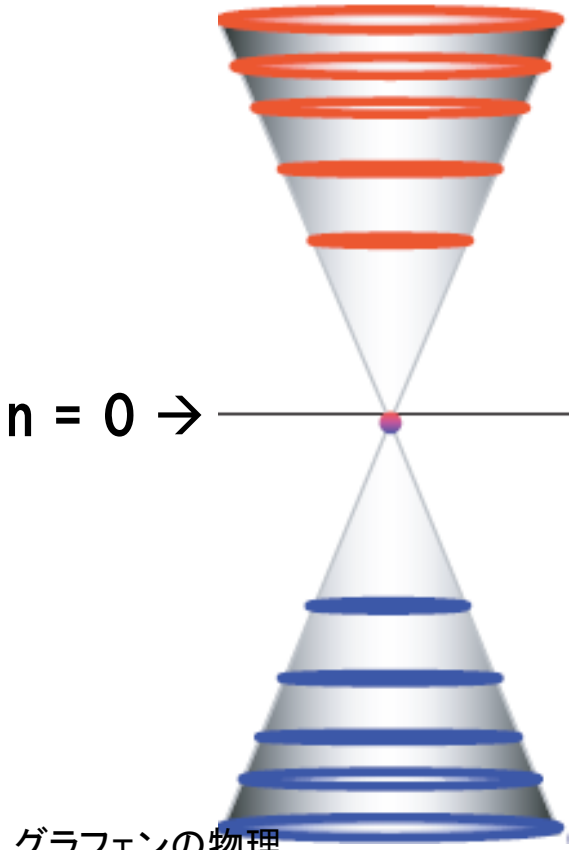
青木 秀夫: グラフェンの物理

What's so special about graphene Landau level

Quantisation in ***B***
↓ Onsager's semiclassical
"Landau tube"

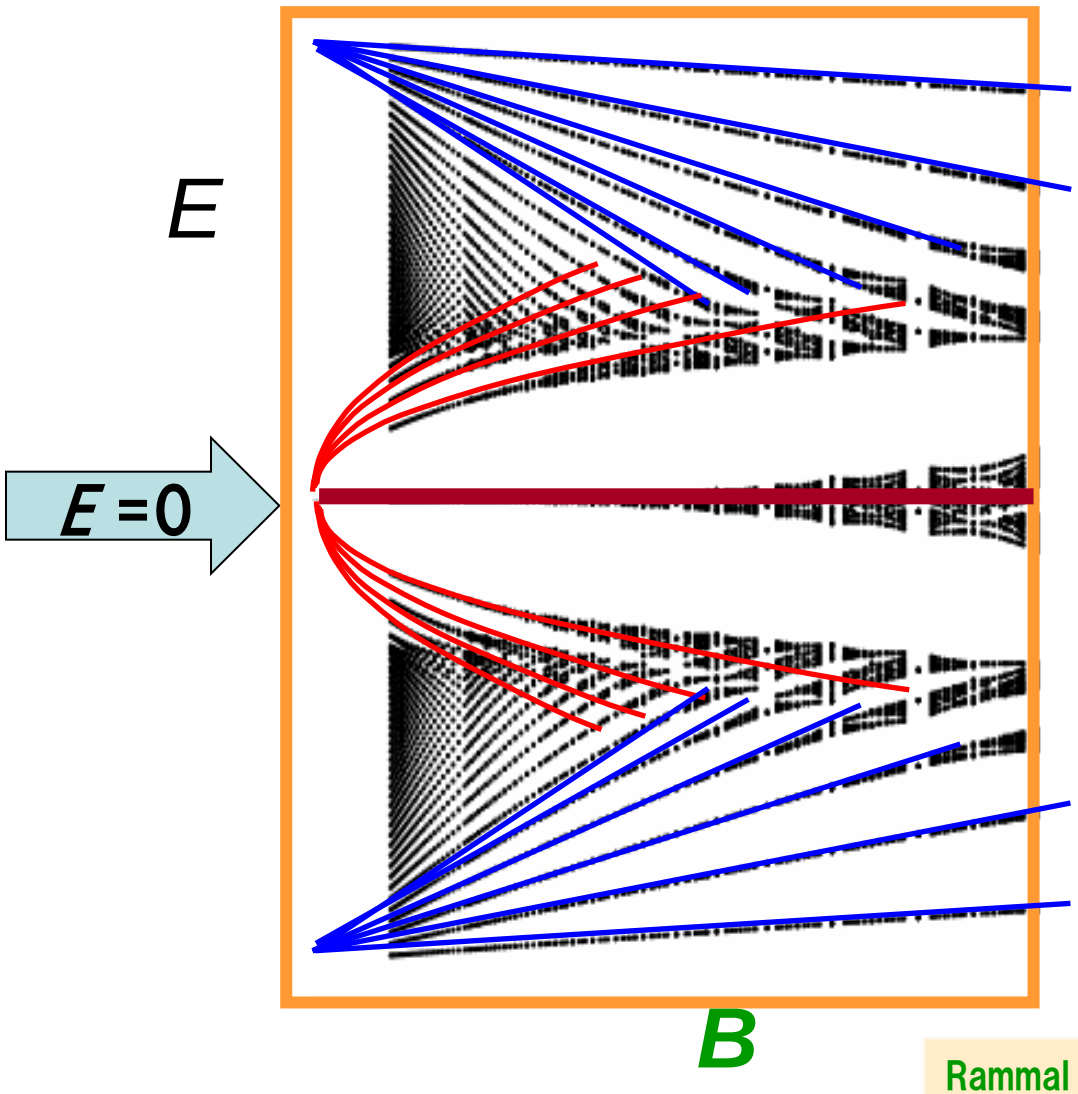


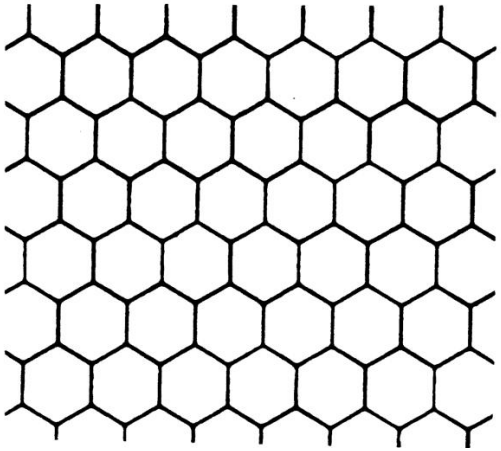
?



青木 秀夫: グラフェンの物理

Whole spectrum for honeycomb lattice





青木 秀夫: グラフェンの物理

Rammal 1985
E =0 Landau level ← outside Onsager's semiclassical quantisation scheme

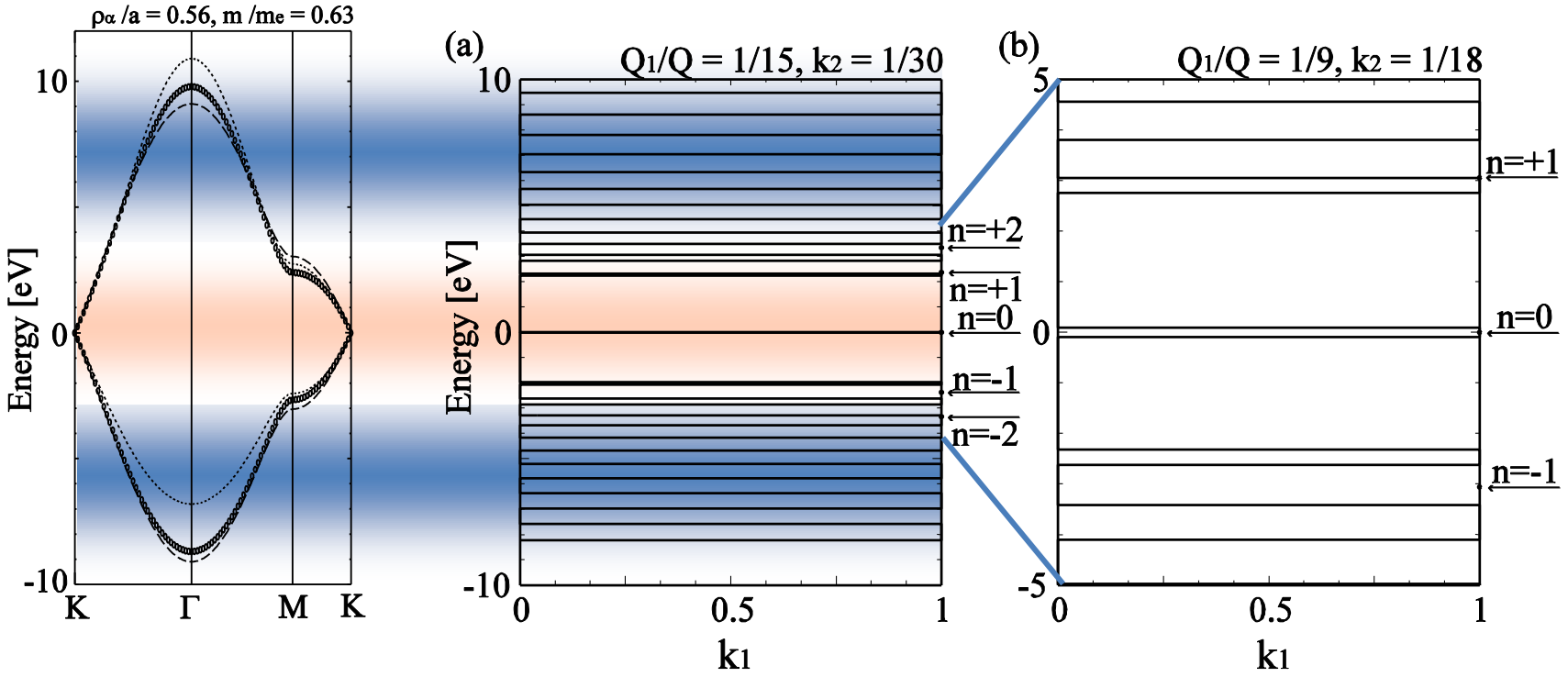
Effect of *B*

* Tight-binding model: Peierls substitution

$$\varepsilon(\mathbf{k}) \rightarrow \varepsilon\left(-i\nabla - \frac{e}{c\hbar}\mathbf{A}\right) \quad \text{i.e.,} \quad c_i^\dagger c_j \rightarrow c_i^\dagger c_j \exp\left(\int_i^j \vec{A}\,d\vec{r}\right).$$

- Peierls substitution only takes care of the effect ~ ***A***
  and ignores the effects ~ A^2
  (= diamagnetic shift and shrinkage of wavefunctions)

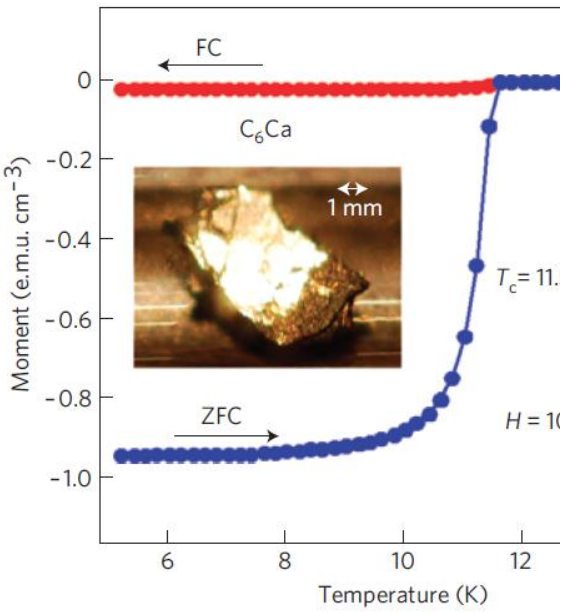
* Honeycomb array of atomic potential + ***B*** (Nakajima & Aoki, Physica E 2008)

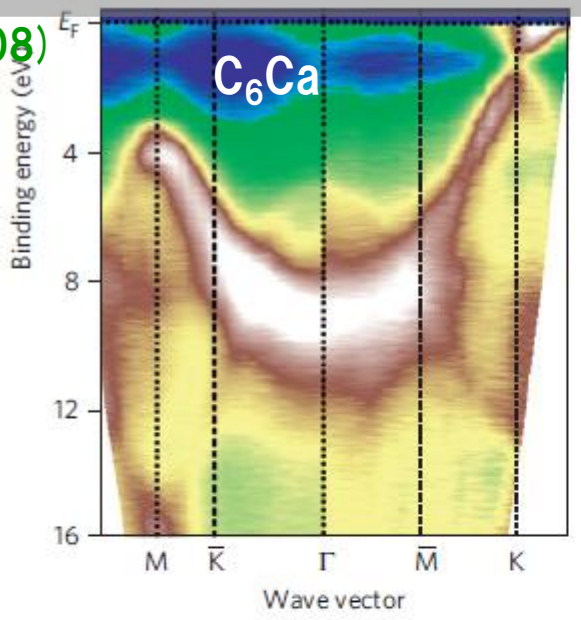


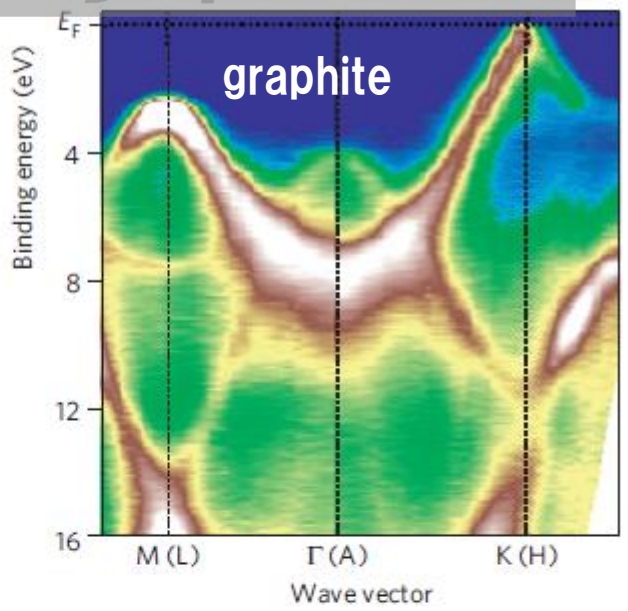
実験

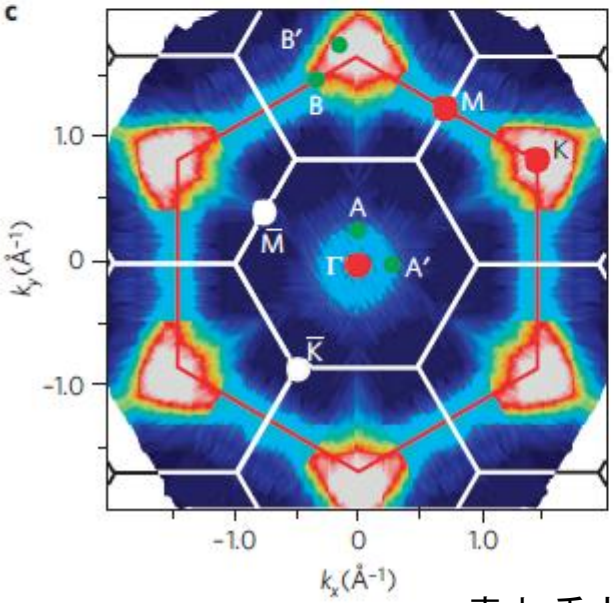
ARPES band structures: GIC vs graphite

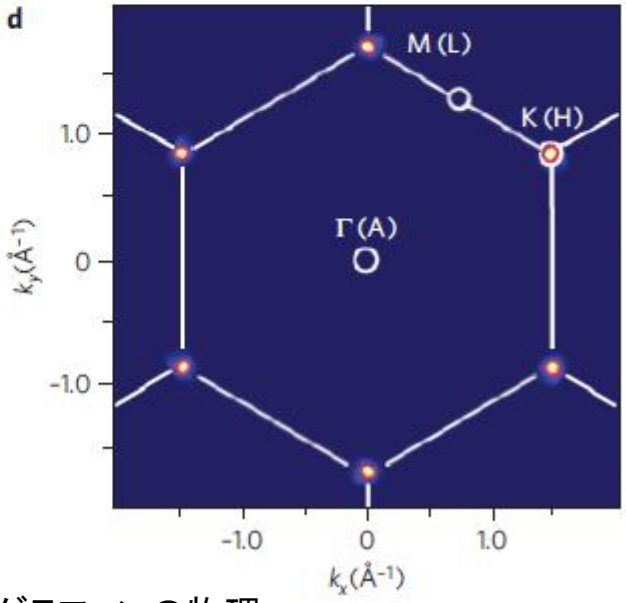
(Sugawara et al, nature phys 2008)





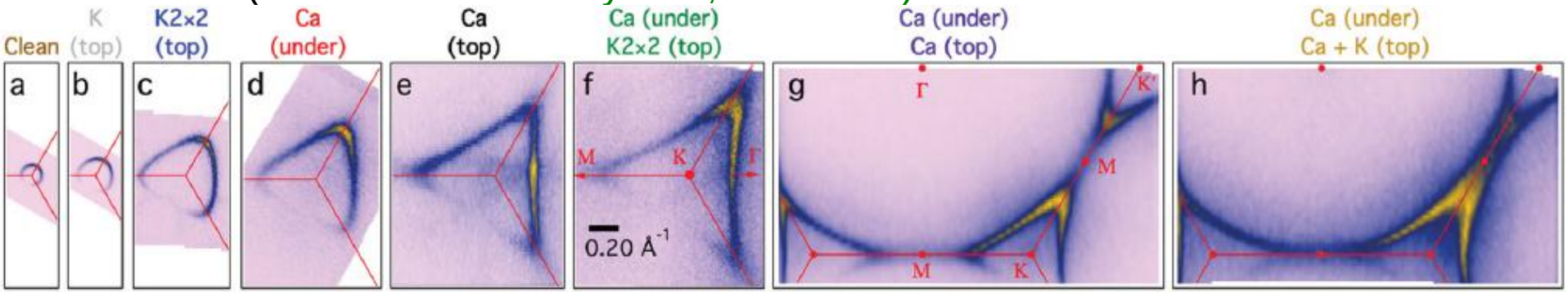




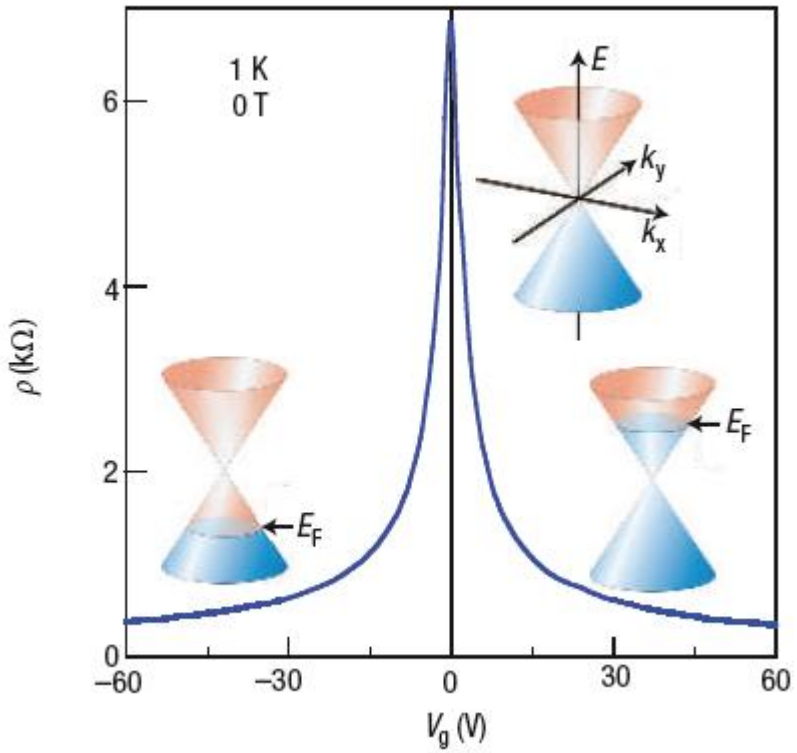


Can graphene be doped?

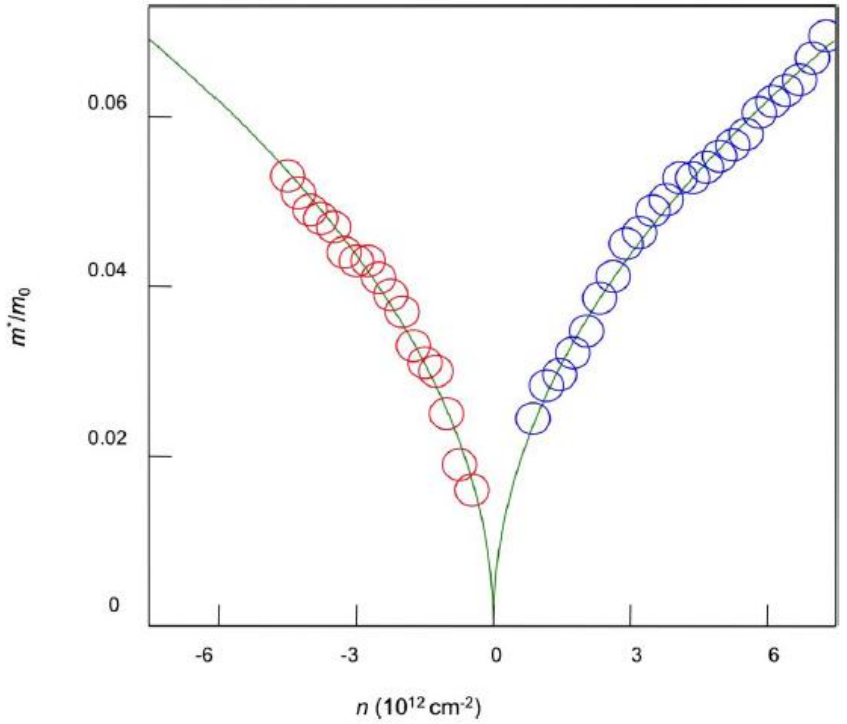
High doping by graphene/SiC+ K, Ca doping
(ARPES: McChesney et al, PRL 2010)



Resistivity　　Cyclotron mass



(Geim & Novoselof, Nature mat. 2007)

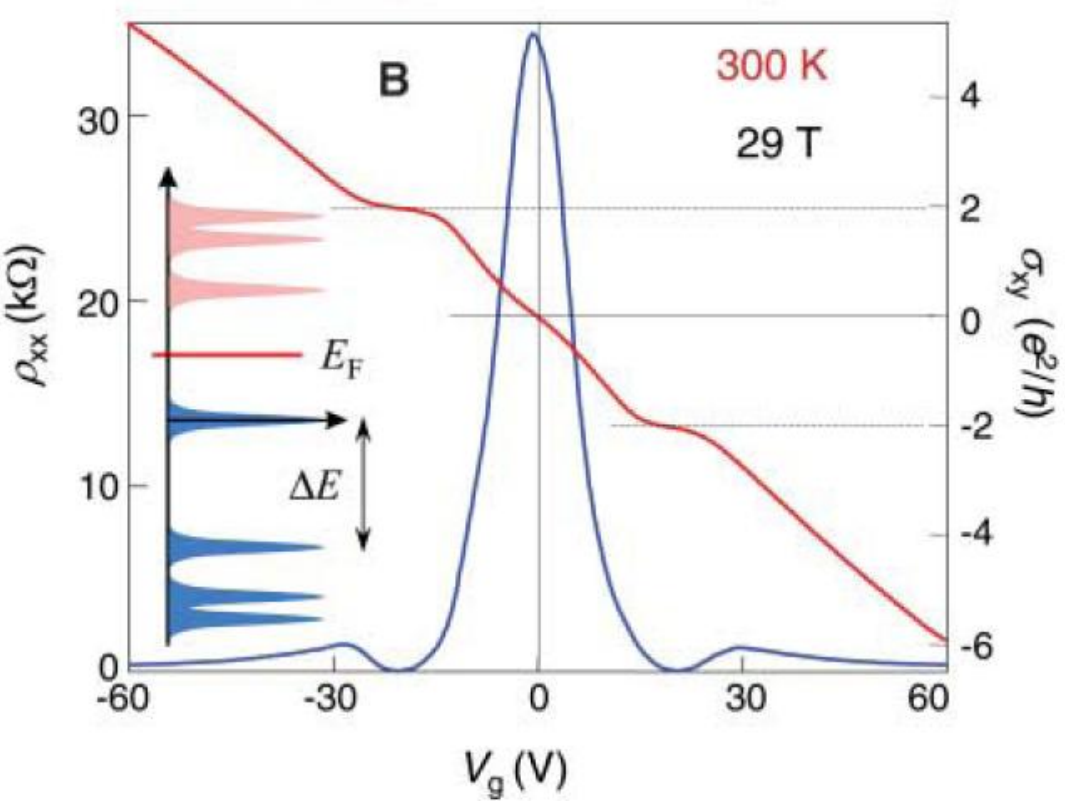


(Castro Neto et al, RMP 2009)

Dirac pt has no electrons, no holes, yet σ ¥neq 0
← Interband phenomenon (or "Zitterbewegung")

$n = (\hbar/2\pi e)S$, $m_c = (\hbar^2/2\pi)\partial S/\partial E \sim n^{1/2}$
→ $S \sim E^2$ (***k***) ie, $E \sim k$

Room-temperature QHE

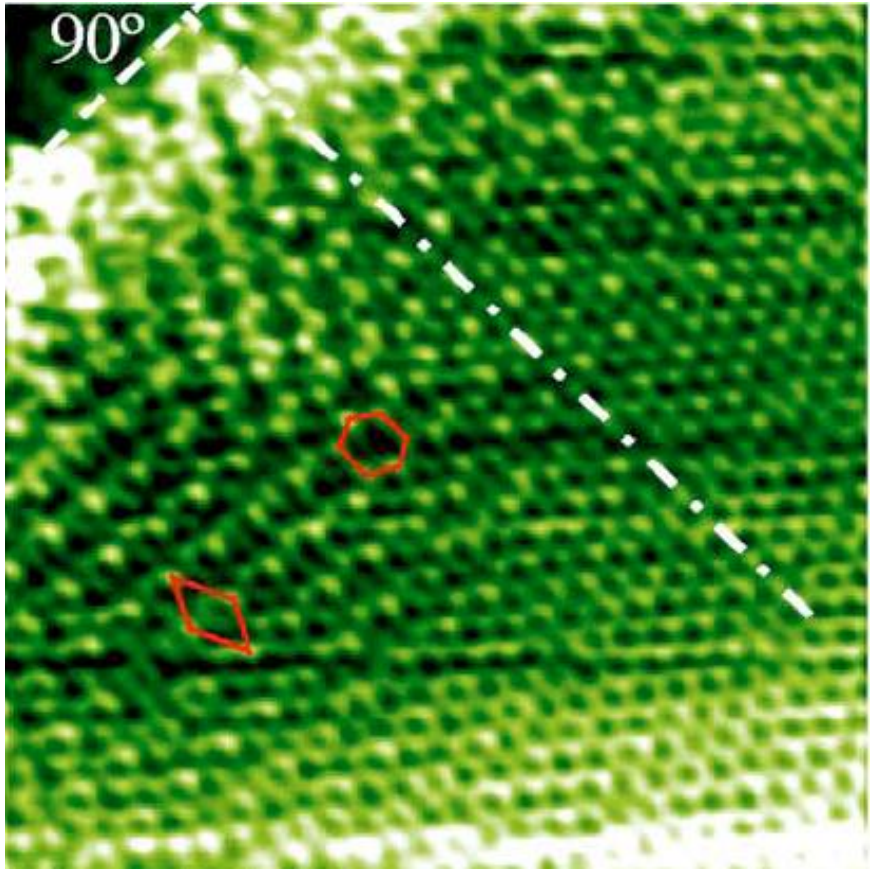


青木 秀夫: グラフェンの物理

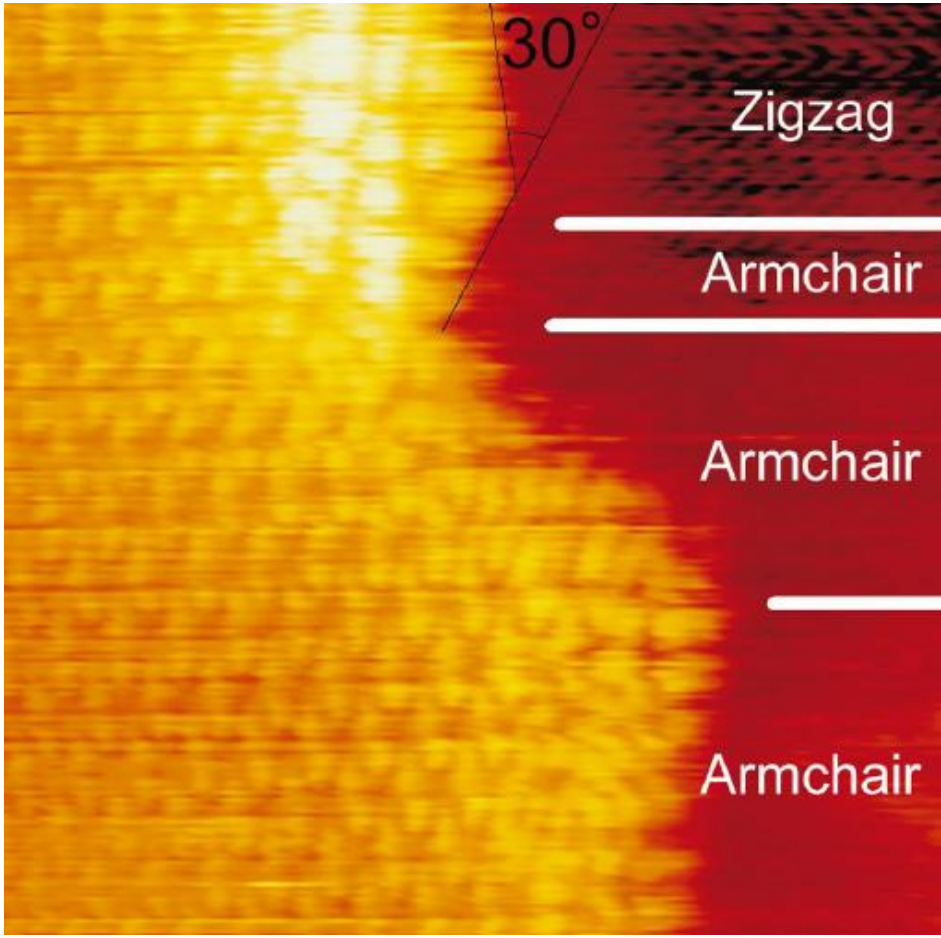
(Novoselof et al, Science 2007)

STM

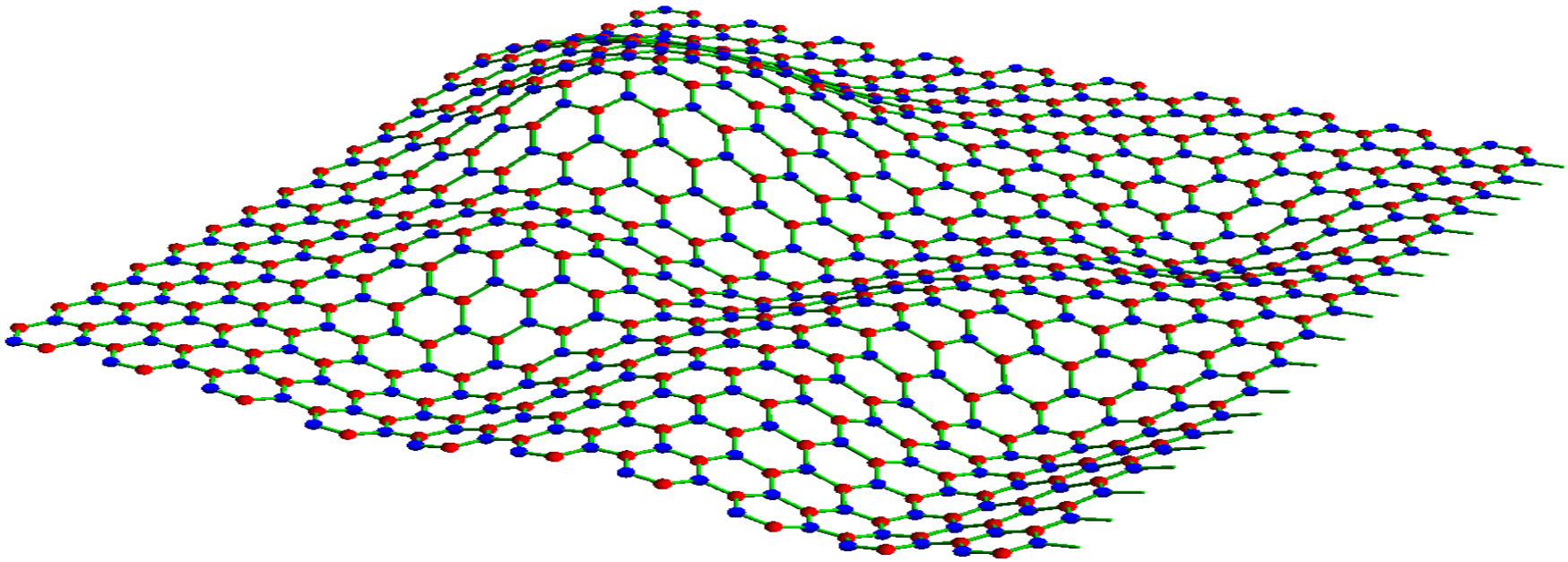
Graphite edge (Niimi et al, PRB 2006)



Graphene edge (Kobayashi et al, PRB 2005; 榎ほか、固体物理 2010))



Ripple



Textbook on statistical mechanics → 2D crystals cannot exits

→ why does graphene exist? → ripple

More recently: Higgs-mechanism-induced rippling? (San-Jose et al, PRL 2011)

$$F[u,h]=\frac{1}{2}\int d^2\vec{x}[\kappa(\nabla^2 h)^2+2\mu u_{ij}^2+\lambda u_{ii}^2]$$

bending energy $\kappa \approx 1.1$ eV,

bulk modulus $\lambda+\mu \approx 7.3$ eV Å^{-2},

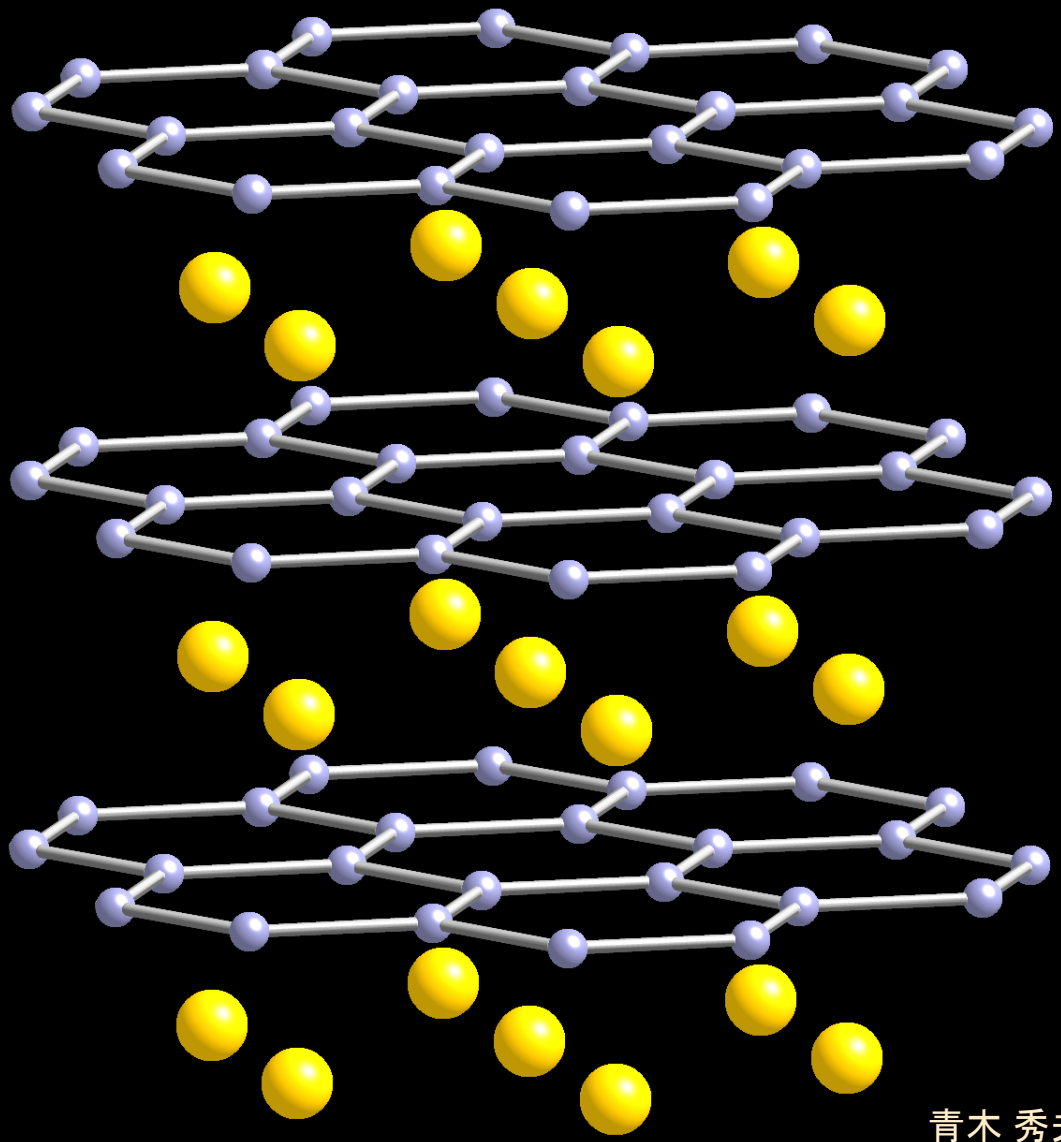
shear modulus $\mu \approx 5.7$ eV Å^{-2}

strain tensor, $u_{ij}=\frac{1}{2}(\partial_i u_j+\partial_j u_i)+\frac{1}{2}(\partial_i h)(\partial_j h)$

→ a relativistic scalar field theory with a Higgs

青木 秀夫: グラフェンの物理

Related structures: GIC, MgB_2



青木 秀夫: グラフェンの物理

MgB_2 bandstructure

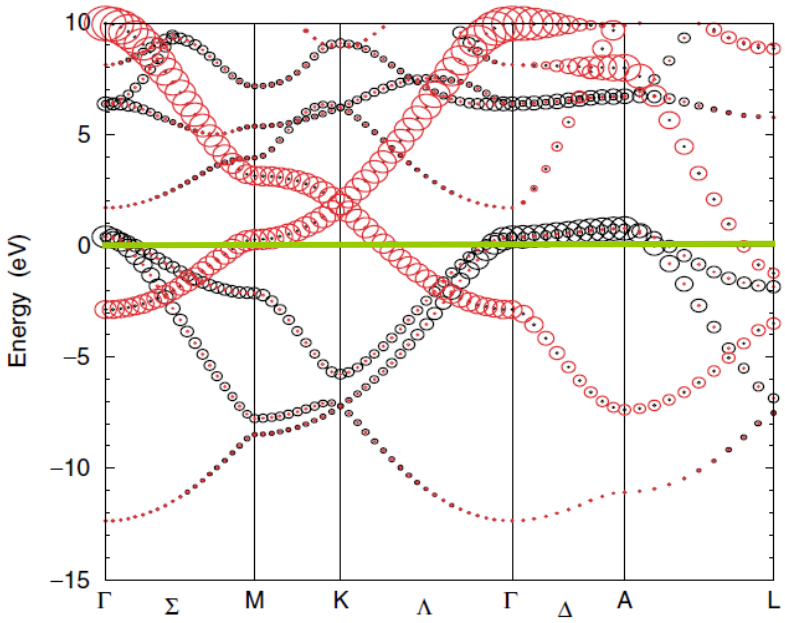
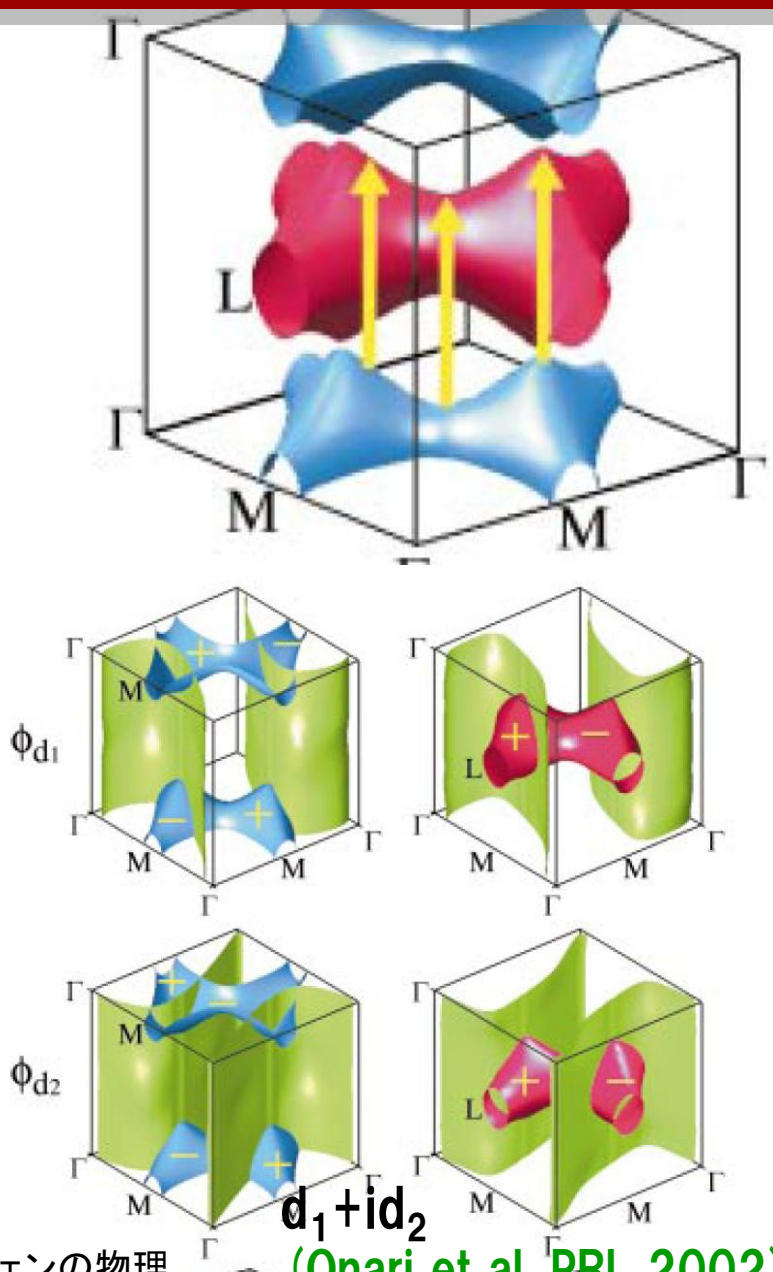


FIG. 1 (color). Band structure of MgB_2 with the B p character. The radii of the red (black) circles are proportional to the B p_z (B $p_{x,y}$) character.

(Kortus et al, PRL 2001)



cf. p wave SC becomes
p + ip at K point in honeycomb
(Uchoa & Castro Neto, PRL 2007)

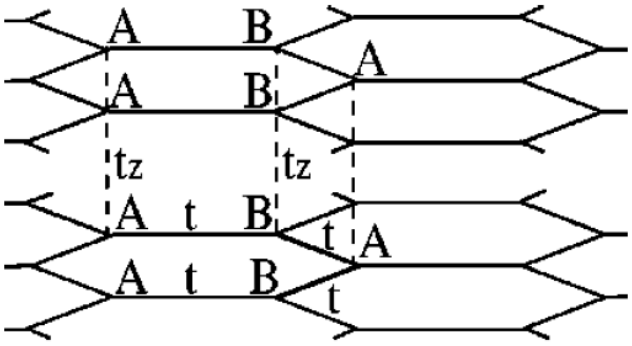
(Onari et al, PRL 2002)

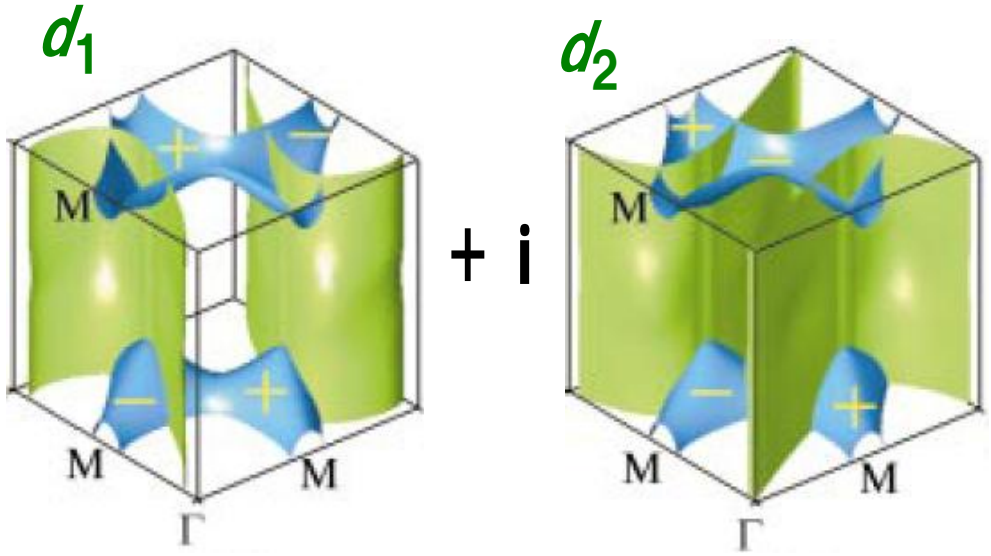
When T-broken pairing can occur?

When the space group of the pair has two-dimensional rep:
as in

- $p + ip$ in tetragonal systems

- $d + id$ in hexagonal systems (Γ_6^+) (Onari *et al*, PRB 2002)
  $p + ip$ (Γ_5^-) (Uchoa & Castro Neto, PRL 2007)





青木 秀夫: グラフェンの物理

QHE in graphene

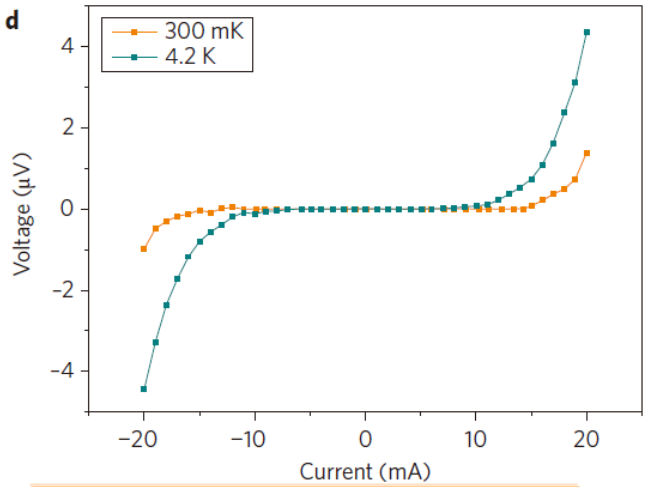
(Novoselov *et al*, Nature 2005; Nature Phys 2006; Zhang *et al*, Nature 2005)

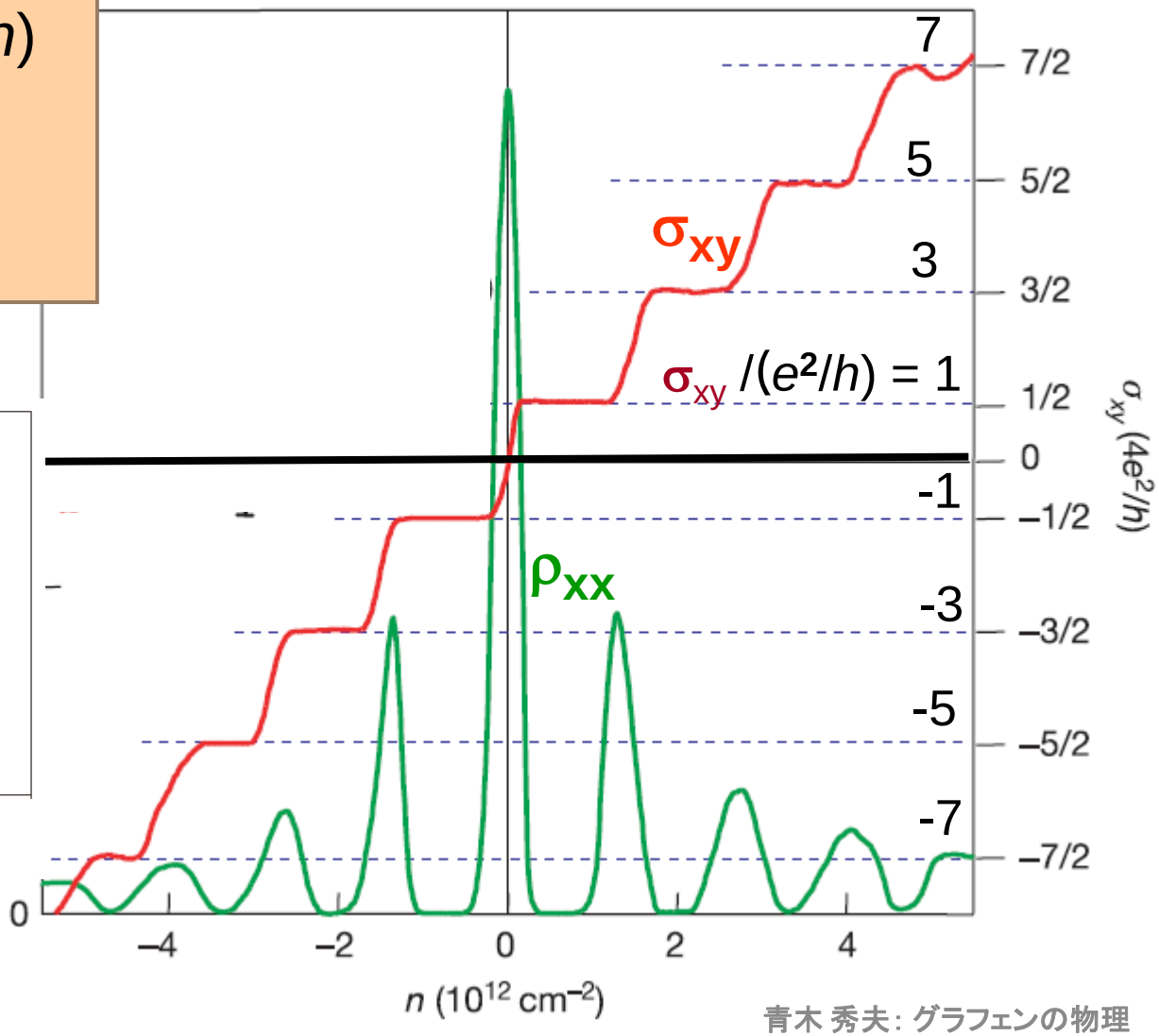
$B = 14\,\mathrm{T}$ and $T = 4\,\mathrm{K}$

$\sigma_{xy} = (2n+1)(-e^2/h)$

$= 2\,(n+1/2)(-e^2/h)$

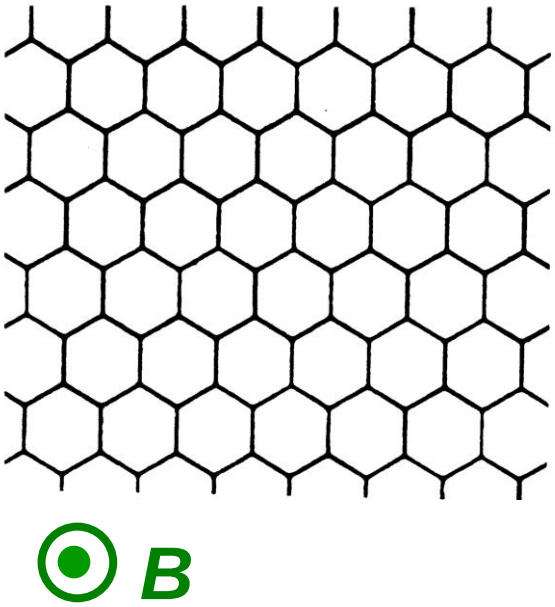
Valley degeneracy



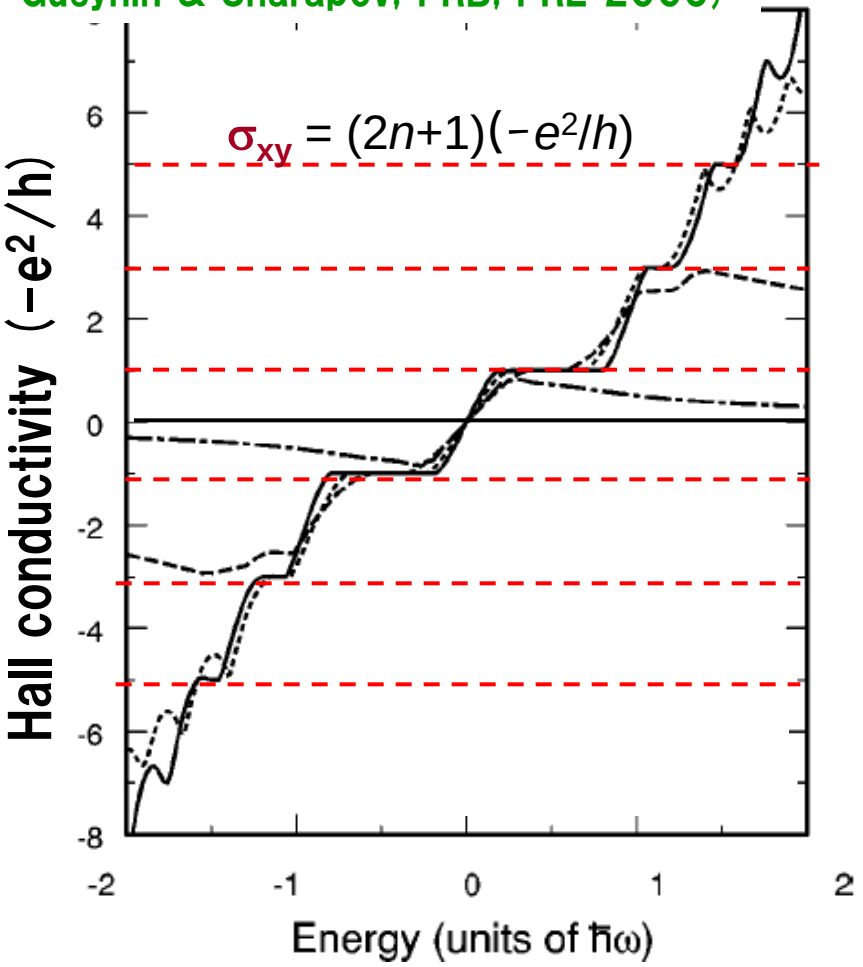


ppb accuracy
(Tzalenchuk et al, nature nanotech 2010)

QHE in honeycomb lattice



(SCBA: Zheng & Ando, PRB 2002;
Gusynin & Sharapov, PRB, PRL 2005)

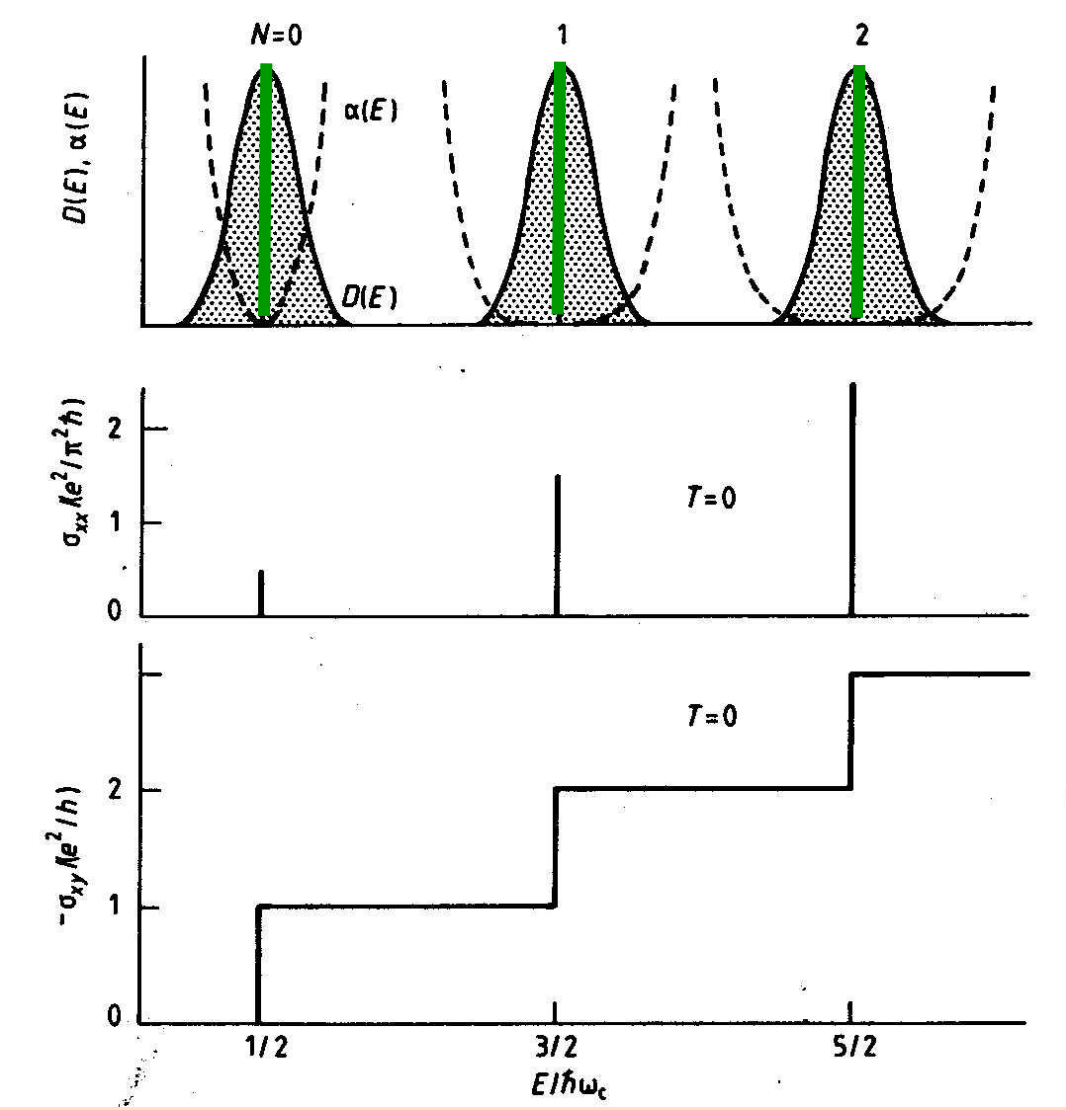


$\sigma_{xy} = (2n+1)(-e^2/h)$ steps

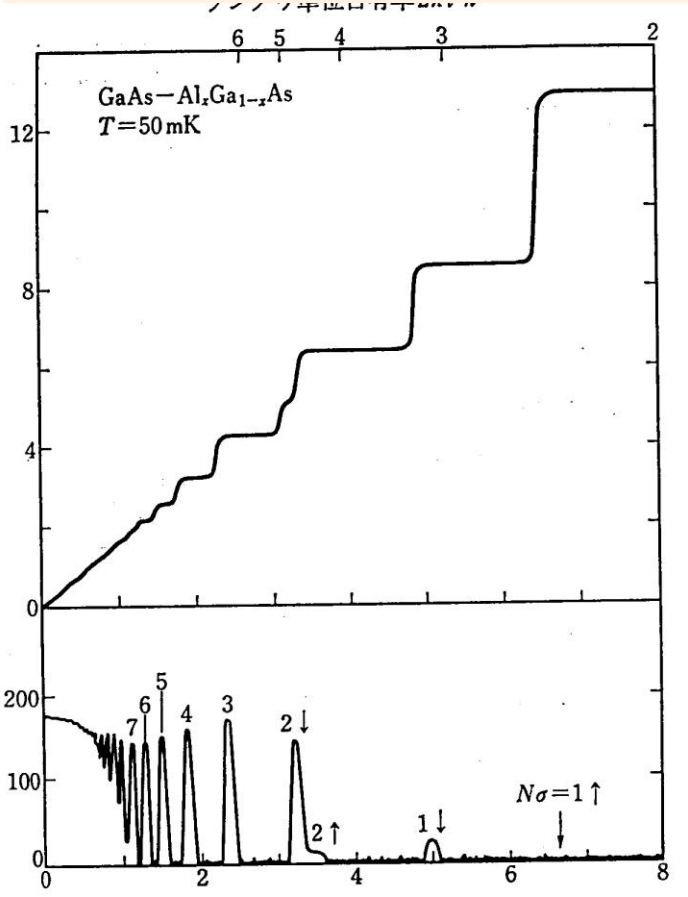
青木 秀夫: グラフェンの物理

Integer quantum Hall effect

(Aoki & Ando 1980)

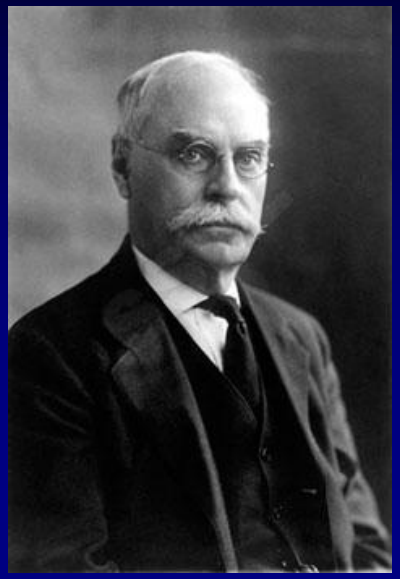


(Aoki, Rep Prog Phys 1987)

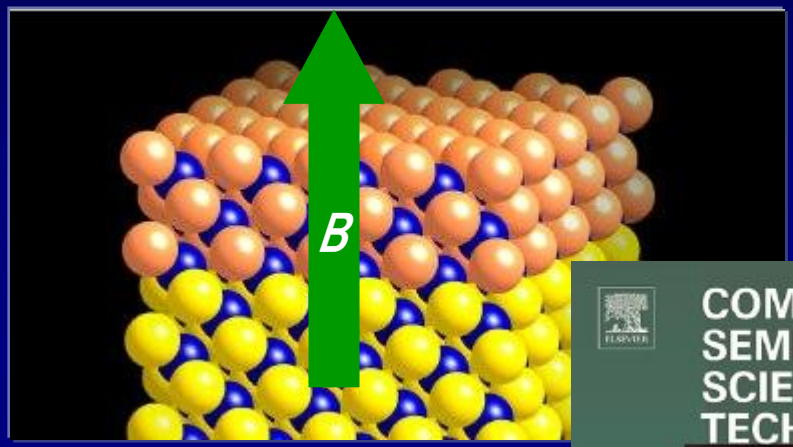


(Paalanen et al, 1982)

2次元電子系



Edwin Hall

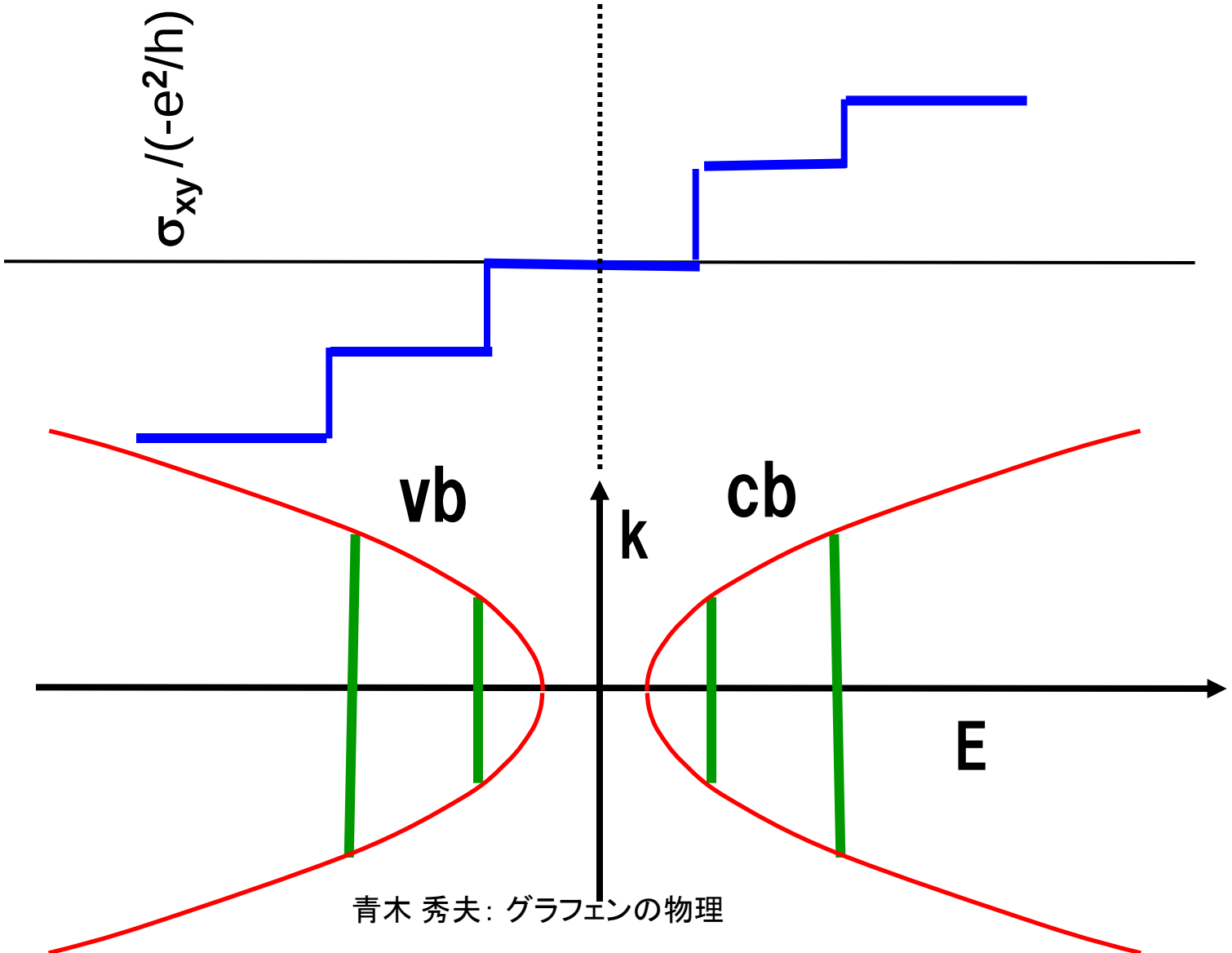


半導体ヘテロ構造



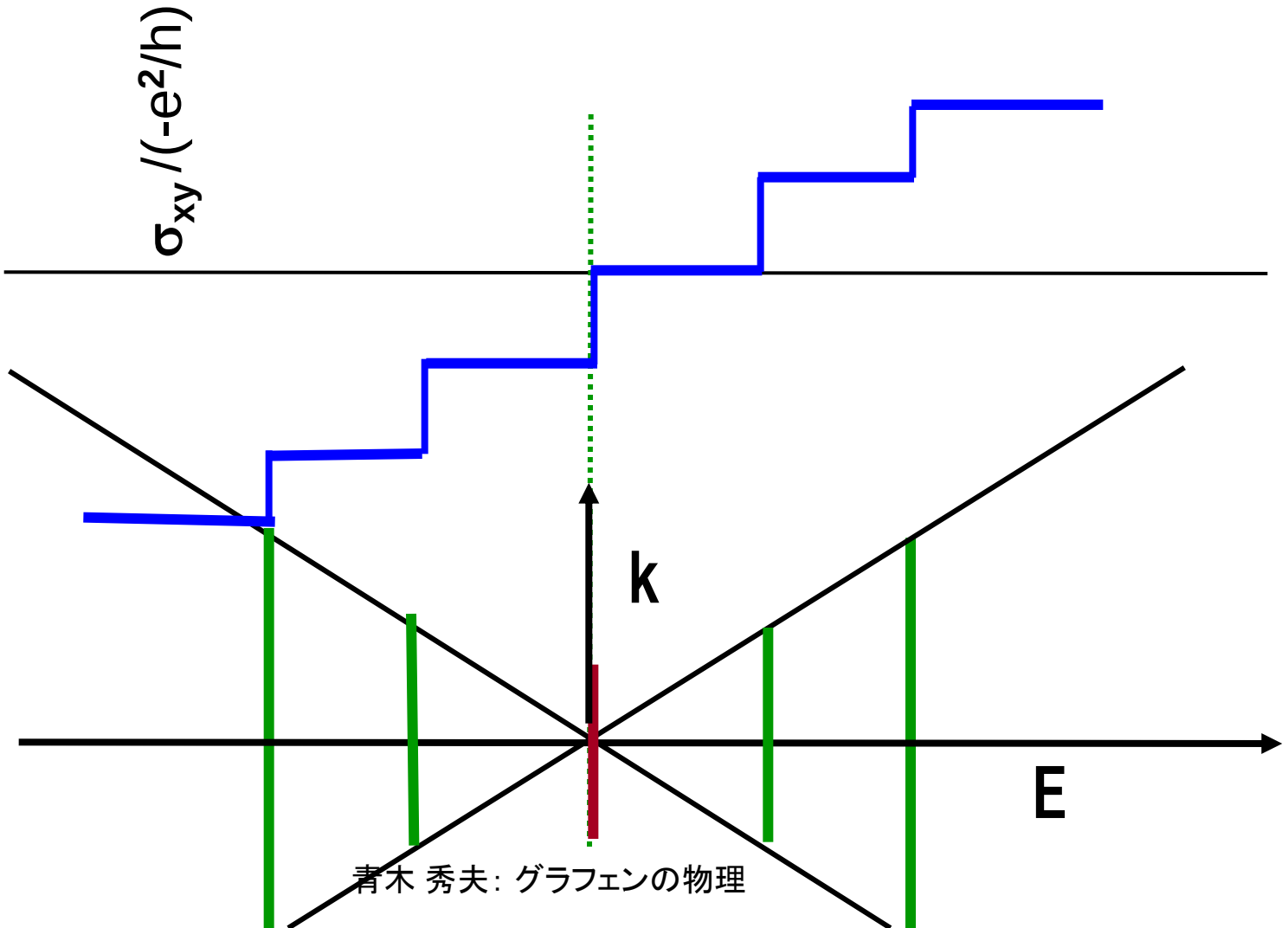
＋ 磁場　→ 量子ホール効果
1991年から、抵抗標準

QHE in ordinary valence / conduction bands



青木 秀夫: グラフェンの物理

QHE for massless Dirac



青木 秀夫: グラフェンの物理

Landau levels for massive Dirac particles

(MacDonald, PRB 1983)

Dirac eqn

$$[(\mathscr{E} - eEx)^2 - (c\vec{p} - e\vec{A})^2 - m^2c^4 + e\hbar cH\ \sigma_z + ie\hbar cE\ \sigma_x]\phi = 0$$

Eigenenergy

$$\mathscr{E}(\vec{k}_\perp, n, \pm) = \frac{H}{\sqrt{H^2 - E^2}} \times \{m^2c^4 + mc^2[\hbar\omega_c(2n + 1 \pm 1)]\}^{1/2}$$

non-relativistic

$$\mathscr{E} = mc^2$$

↓leading term in $\hbar\omega_c/mc^2$ expansion

$$+\hbar\omega_c\left(n + 1 \pm \frac{1}{2}\right) + \frac{m}{2}\left(\frac{cE}{H}\right)^2 + \cdots$$

massless Dirac

?

$\omega_c = eH/mc$

Hall conductivity = a topological number

(Thouless, Kohmoto, Nightingale & den Nijs, 1982)

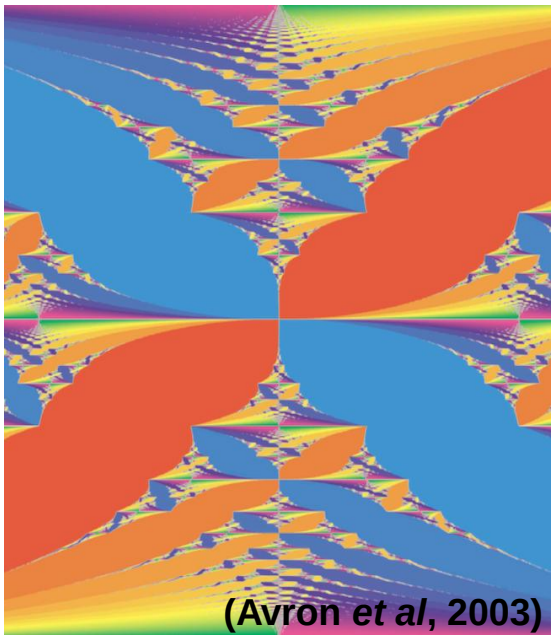
Linear response

$$\sigma_{xy} = -i\frac{1}{L^2}\sum_{\alpha,\beta} f(\varepsilon_\alpha)\frac{\langle\alpha|J_x|\beta\rangle\langle\beta|J_y|\alpha\rangle - \langle\alpha|J_y|\beta\rangle\langle\beta|J_x|\alpha\rangle}{(\varepsilon_\alpha - \varepsilon_\beta)^2}$$

clean, periodic systems

Berry's "curvature"

$$= -i\frac{e^2}{L^2}\sum_{n,\vec{k}} f(\varepsilon_{n\vec{k}})\left[\vec{\nabla}_{\vec{k}} \times \left\langle n\vec{k}\middle|\vec{\nabla}_{\vec{k}}\middle|n\vec{k}\right\rangle\right]_z$$



(Avron *et al*, 2003)

"Gauss-Bonnet"

$= \sum_n^{\text{band}} (\text{Chern } \#)_n$

disordered systems
distribution of topological #
in disordered systems
(Aoki & Ando, 1986;
Huo & Bhatt, 1992;
Yang & Bhatt, 1999)

Hall conductivity

Chern number $\sigma_{xy}(E) = \frac{e^2}{h} C_E$ $C_E = \frac{1}{2\pi i} \int_{T^2} \mathrm{Tr}_M \, dA$ $A_{ij} = \langle \psi_i | d\psi_j \rangle$

M × M Matrix

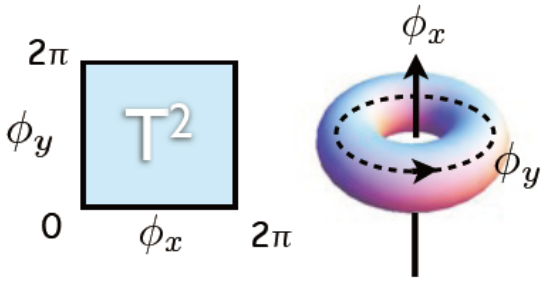
Filled states

Thouless, Kohmoto, Nightingale, den Nijs (1982)
Aoki, Ando (1986), Niu, Thouless, Wu (1985)

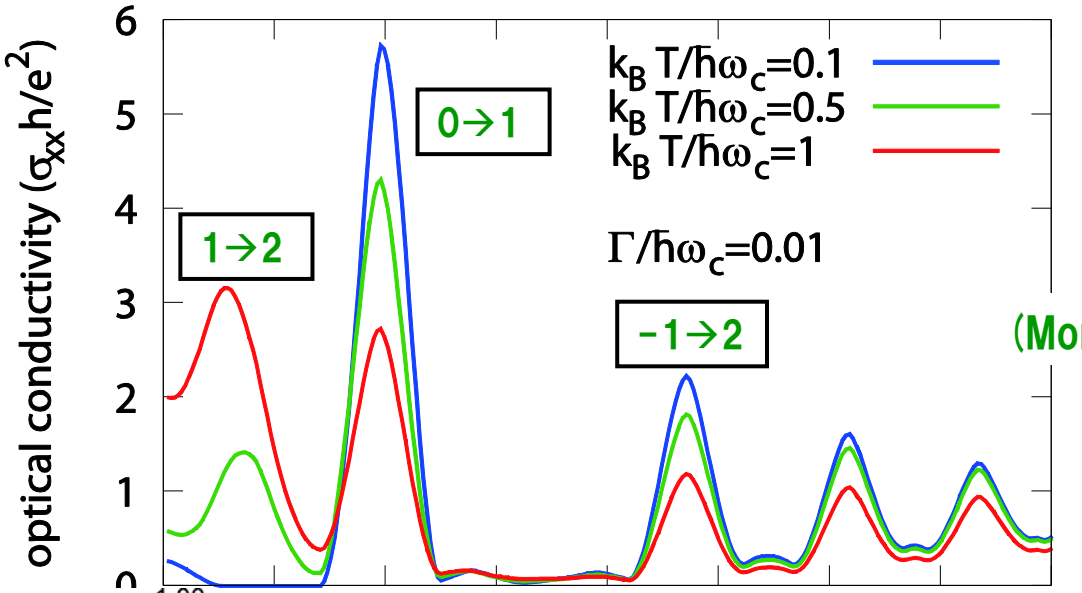
Recent technical development lattice gauge technique

Hatsugai (2004,2005), Fukui, Hatsugai, Suzuki (2005)

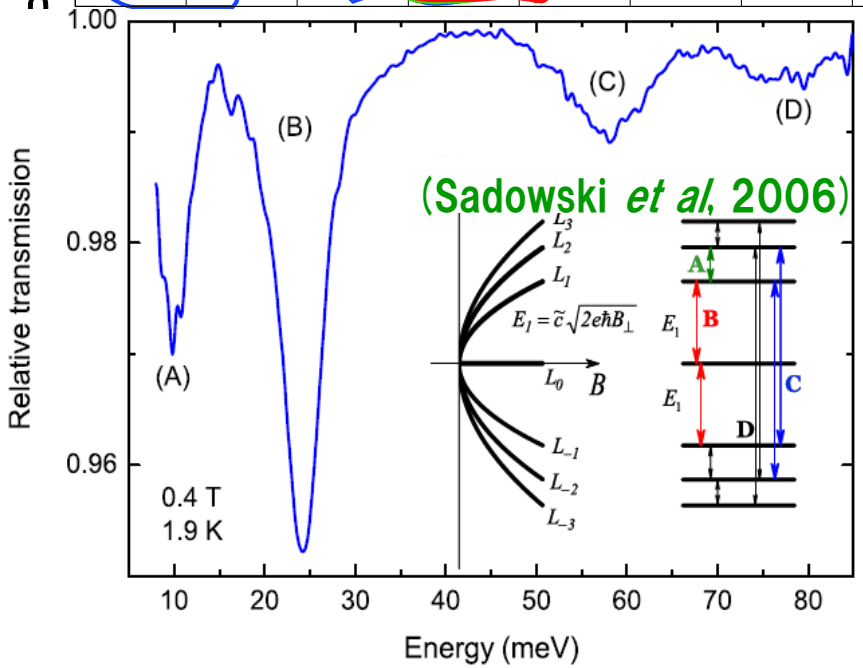
Song, Maruyama, Hatsugai (2007) *Hatsugai, Fukui, Aoki (2006)*



Optical conductivity in graphene (in equilibrium)



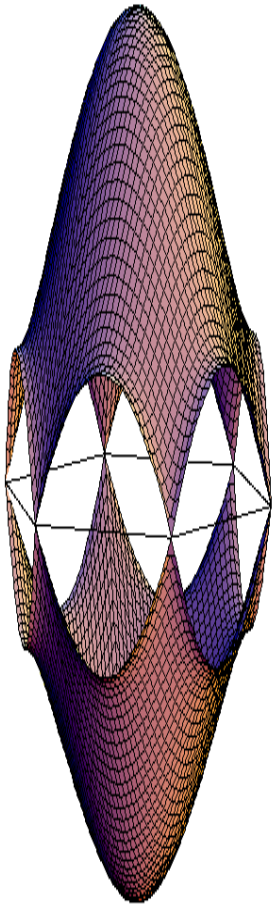
(Morimoto *et al*, 2008)

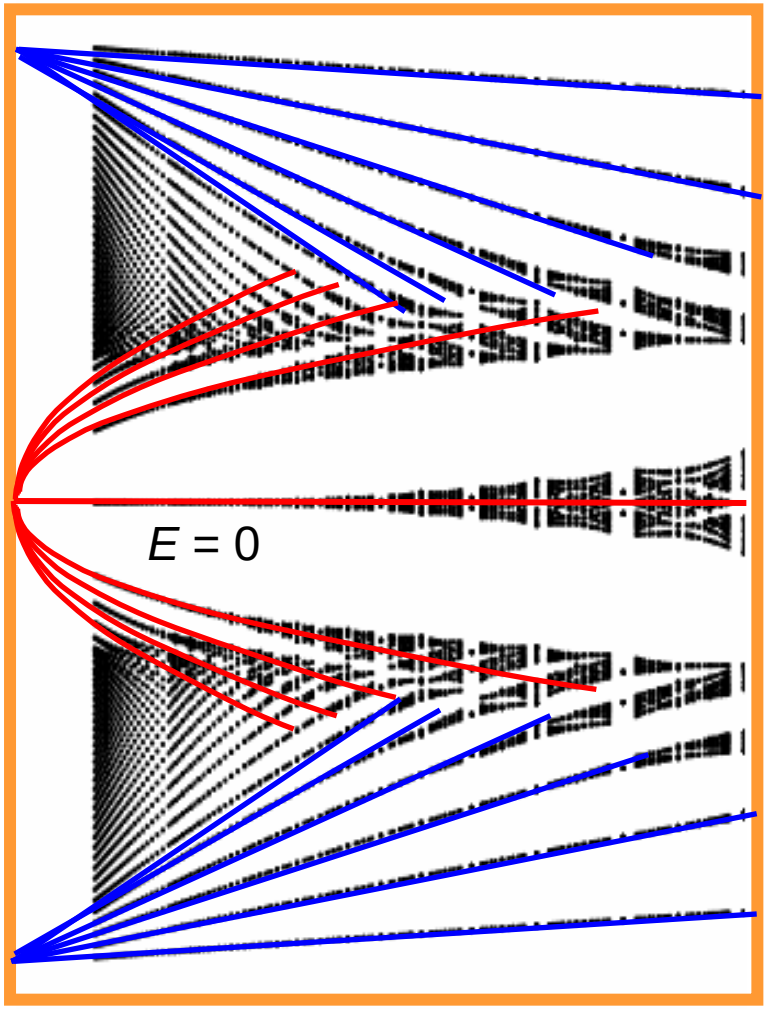


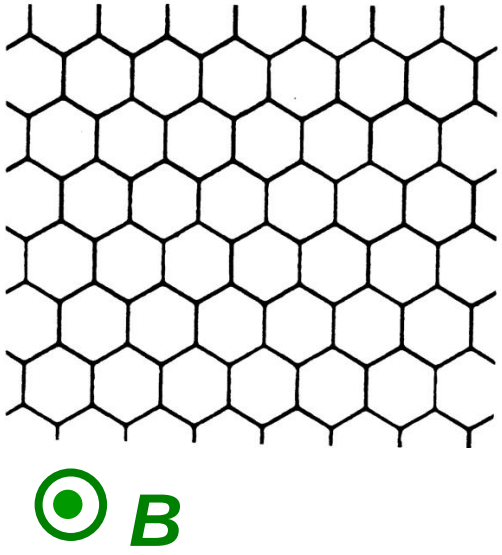
(Sadowski *et al*, 2006)

蜂の巣格子の理論的側面

Hofstadter butterfly for honeycomb lattice

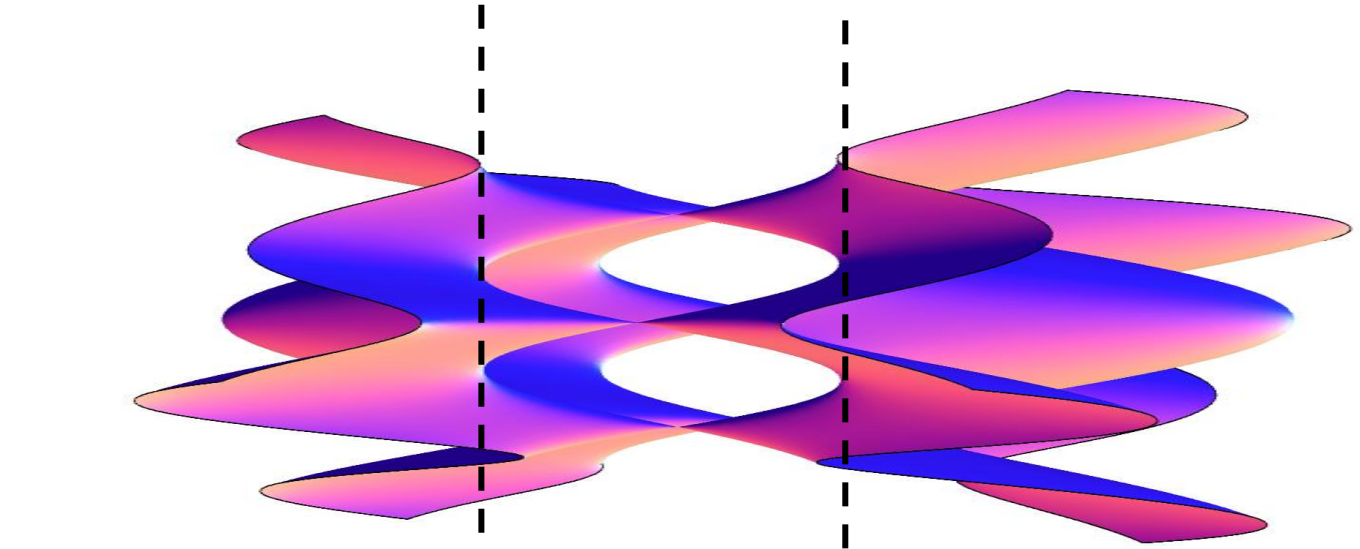


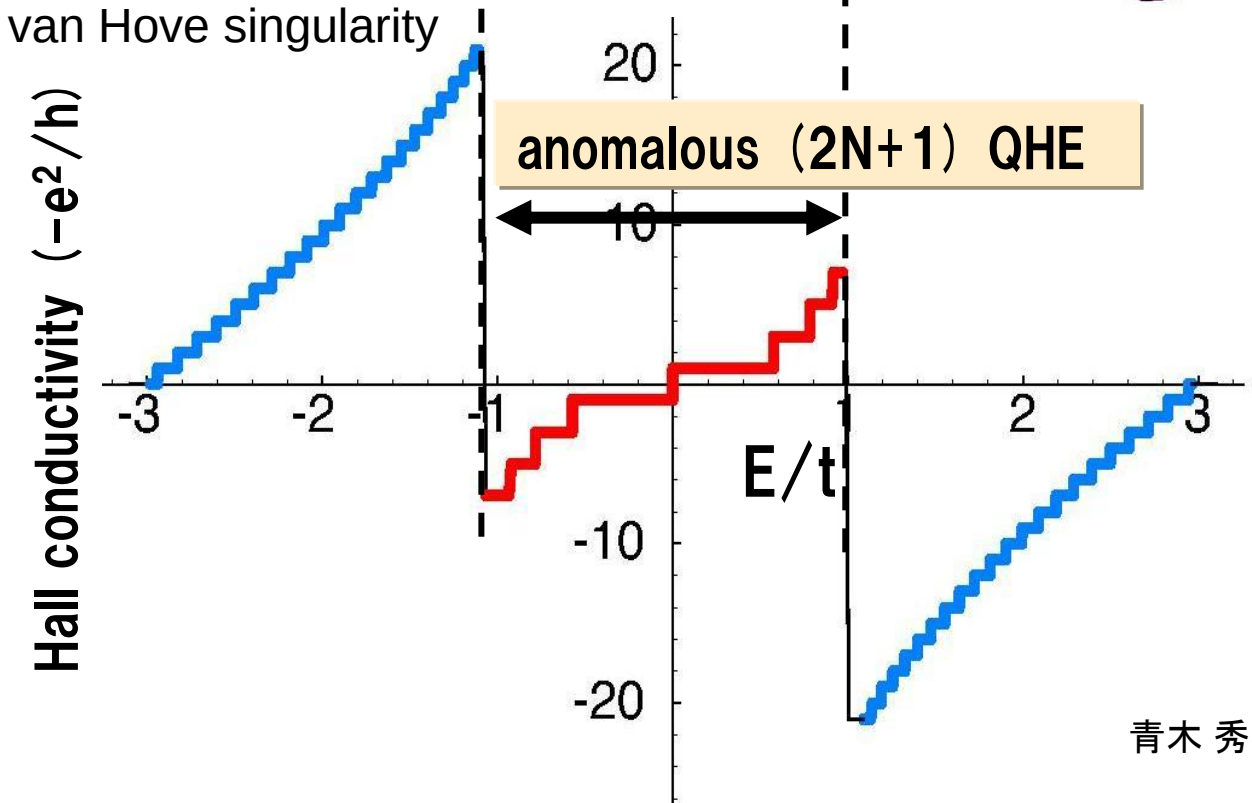




Rammal 1985

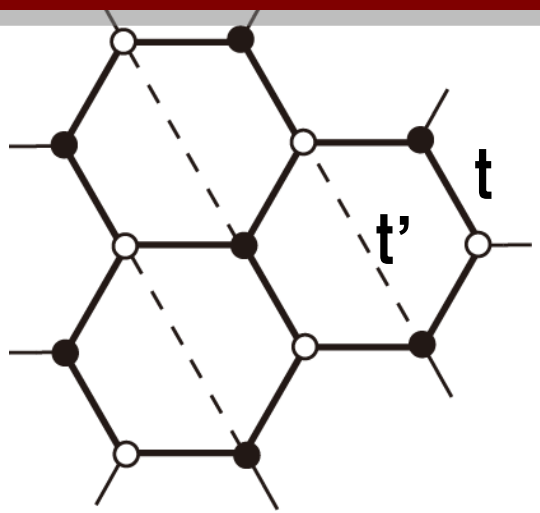
(Hatsugai, Fukui & Aoki, PRB 2006)





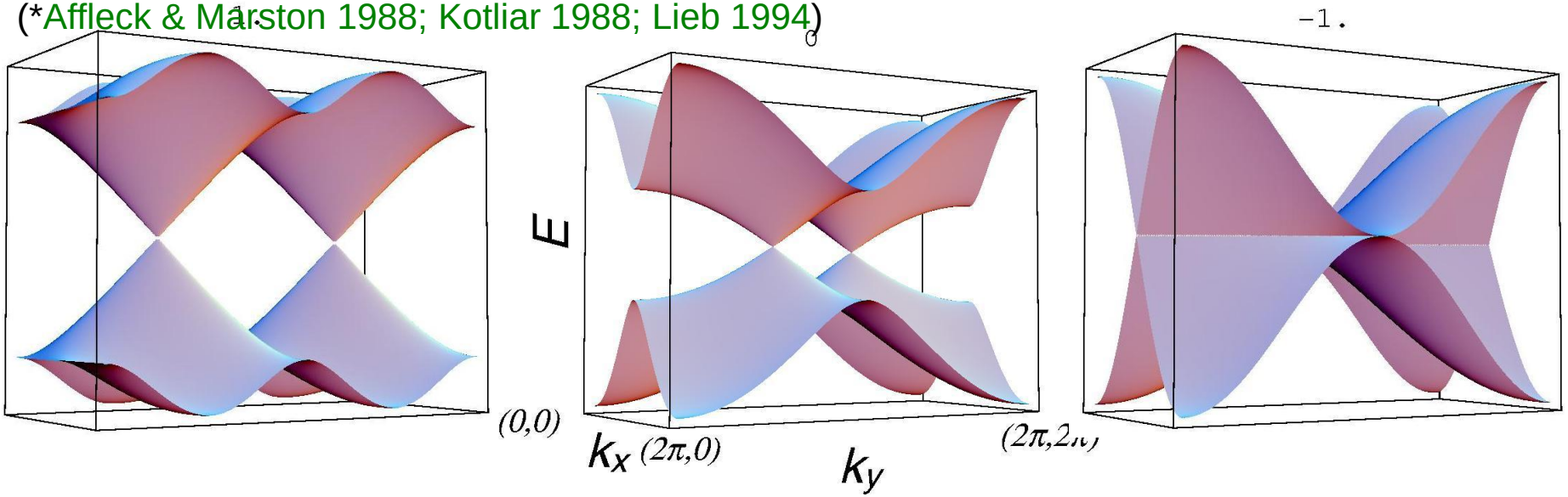
青木 秀夫: グラフェンの物理

"Massless Dirac" sequence

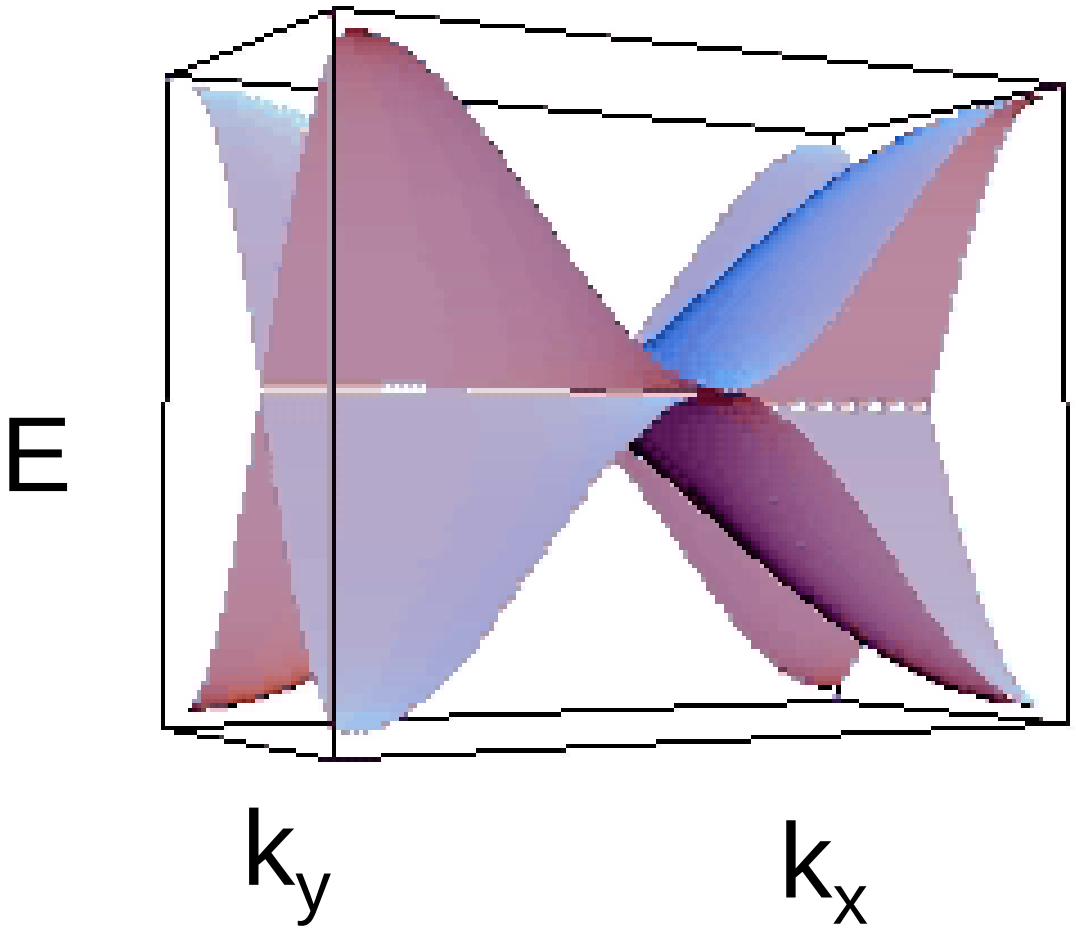


t'=−1: π−flux* lattice　　t'=0: honeycomb　　t'=+1: square

(*Affleck & Marston 1988; Kotliar 1988; Lieb 1994)



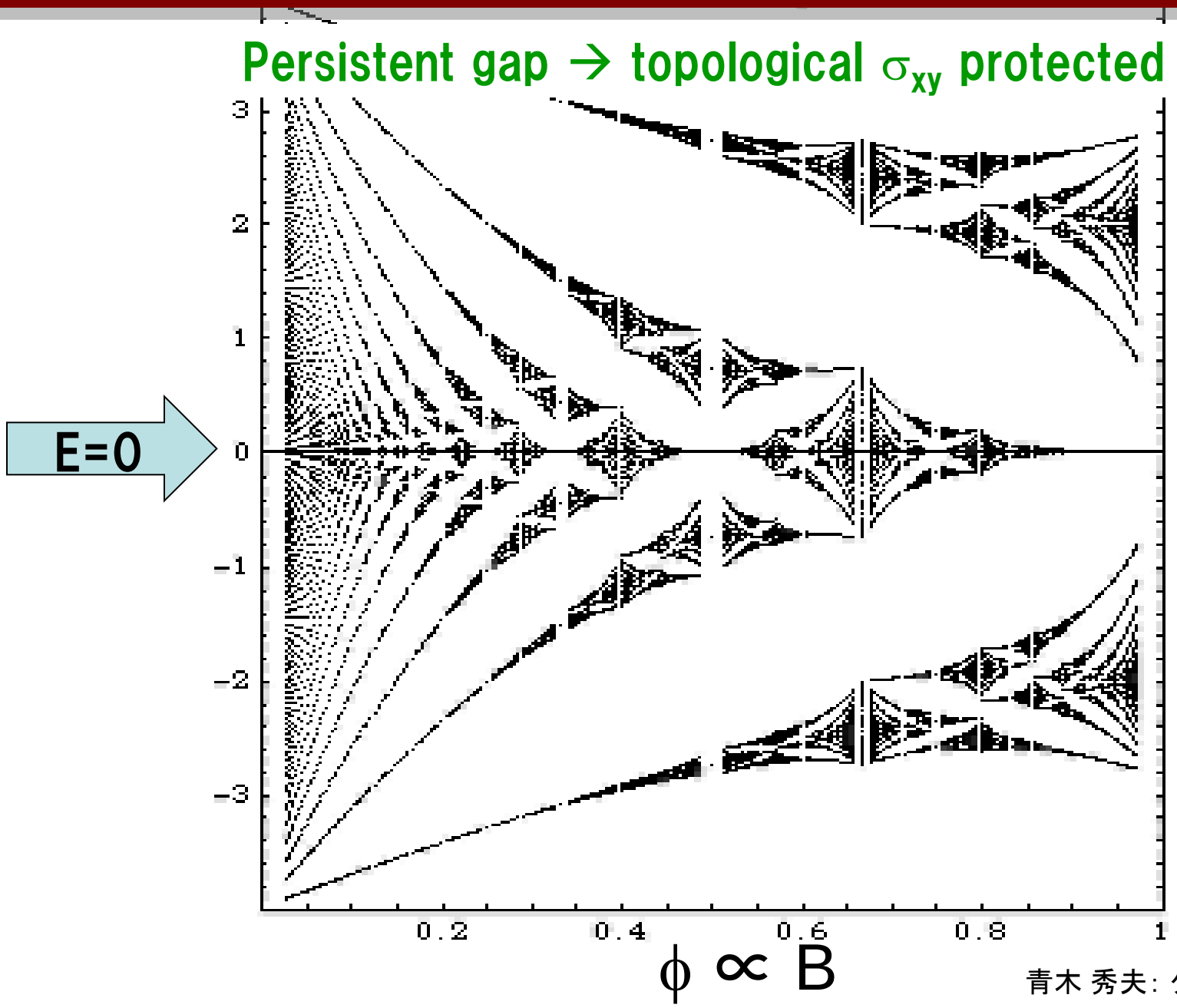
青木 秀夫: グラフェンの物理



* Not specific to graphene
  but shared by a class of non-Bravais lattices
  cf. recent organic system
* Dirac cones appear in pairs
  --- Nielsen-Ninomiya

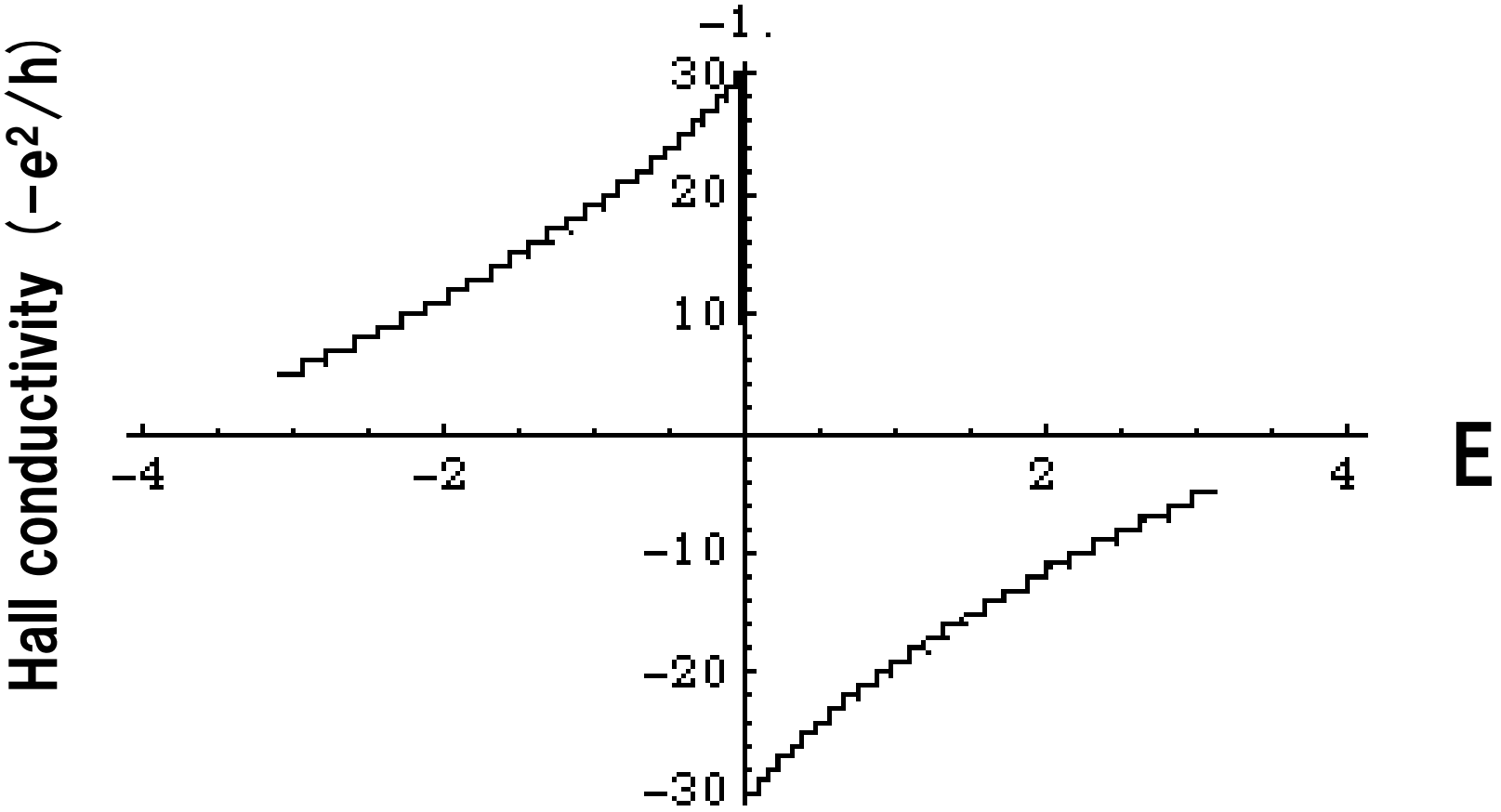
Adiabatic continuity for QHE

Persistent gap → topological σ_{xy} protected



青木 秀夫: グラフェンの物理

QHE topological



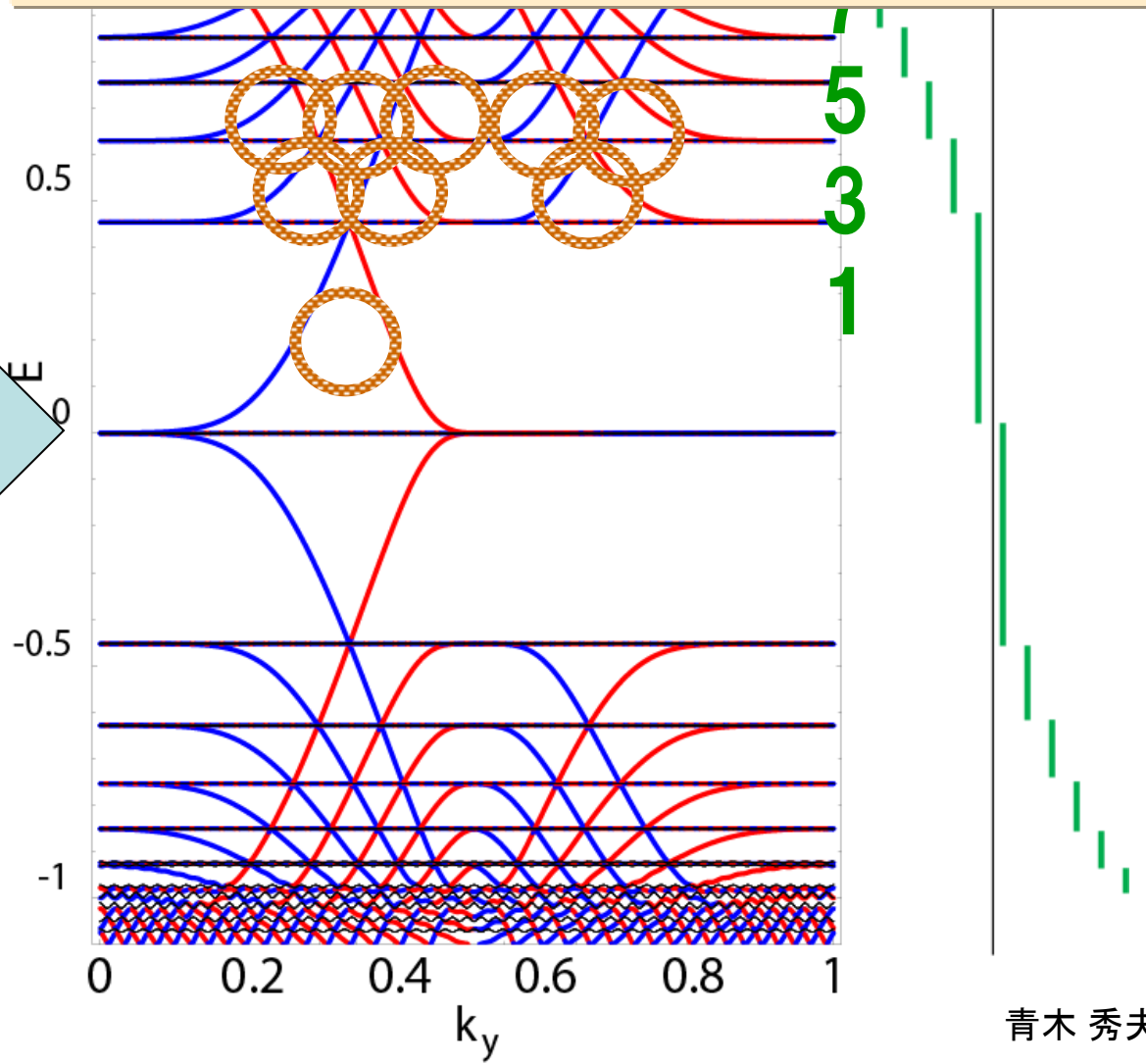
青木 秀夫: グラフェンの物理

$\sigma_{xy}^{bulk} = \sigma_{xy}^{edge}$ in honeycomb

Ordinary QHE: $\sigma_{xy}^{bulk} = \sigma_{xy}^{edge}$ ← Hatsugai, 1993
with Laughlin's argument

honeycomb

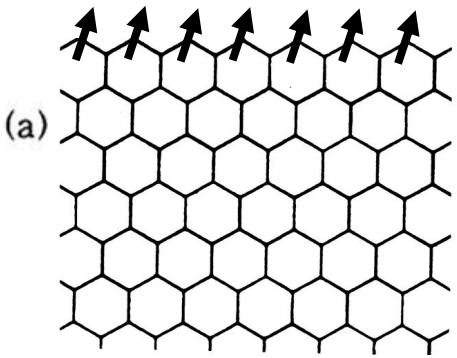
E=0

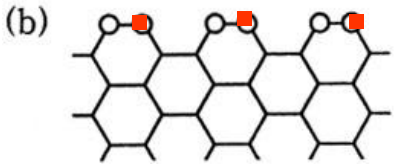


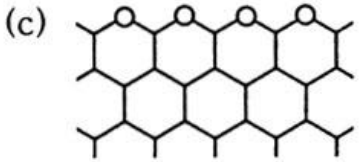
青木 秀夫: グラフェンの物理

Edge states in graphene

Nakata et al, PRB **54**, 17954 (1996)

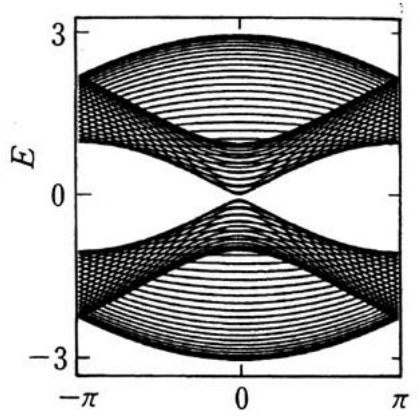


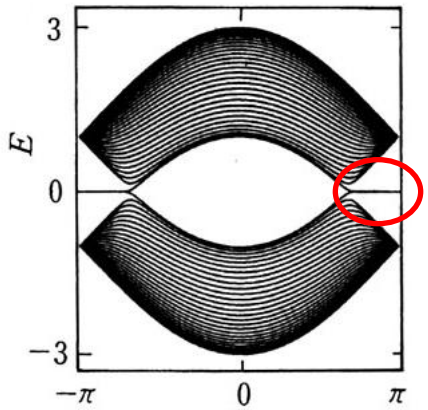




armchair

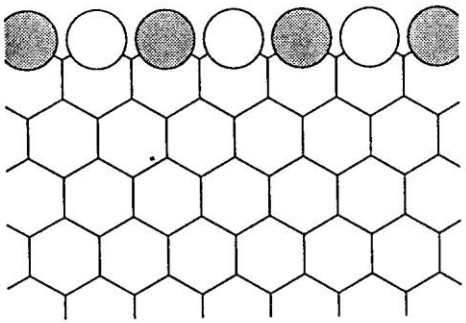
zigzag



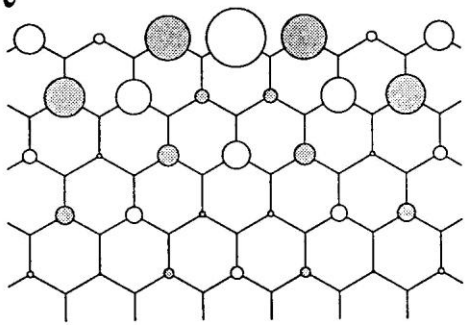


青木 秀夫: グラフェンの物理

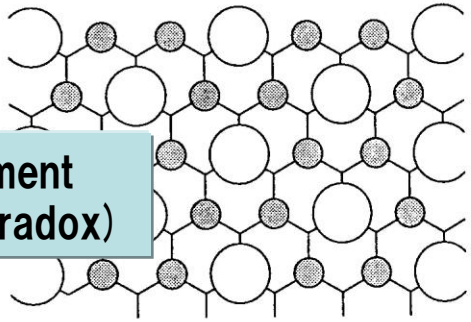
$k=\pi$



$k=\frac{7}{9}\pi$



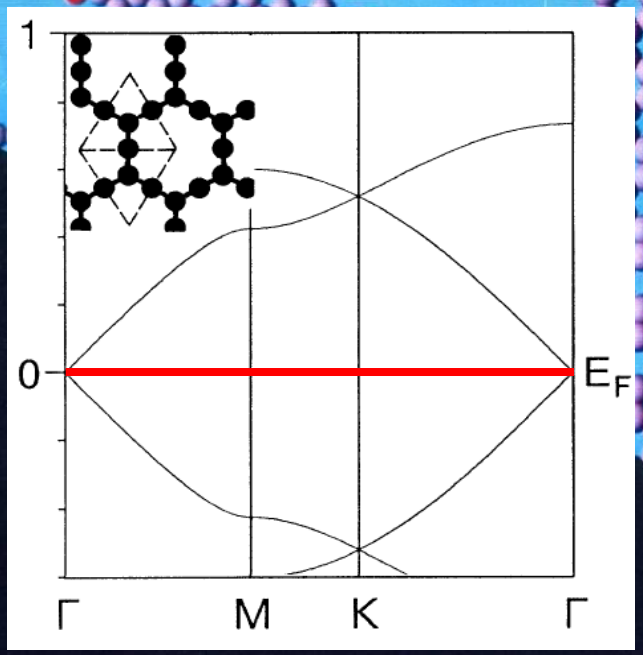
$k=\frac{2}{3}\pi$

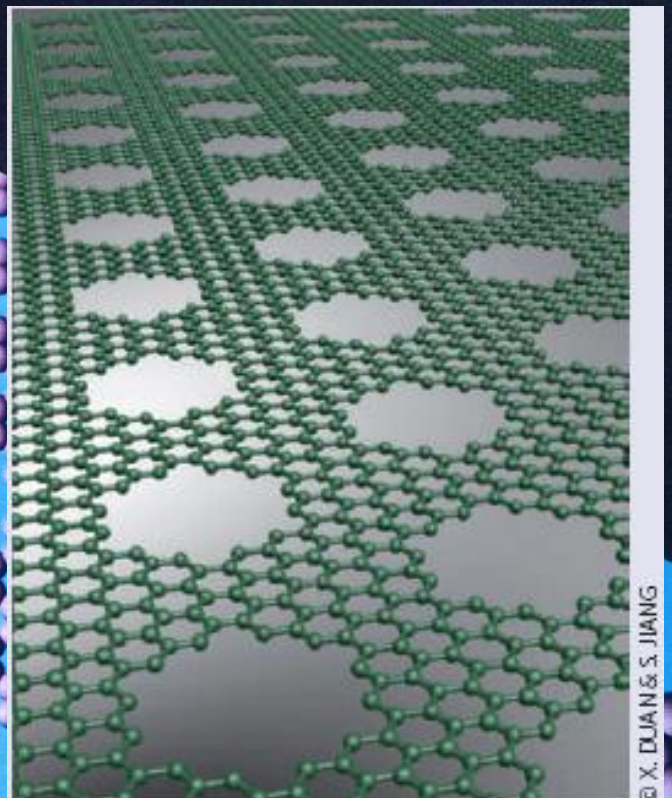


cf. Potential confinement impossible (Klein paradox)

Long-period graphene

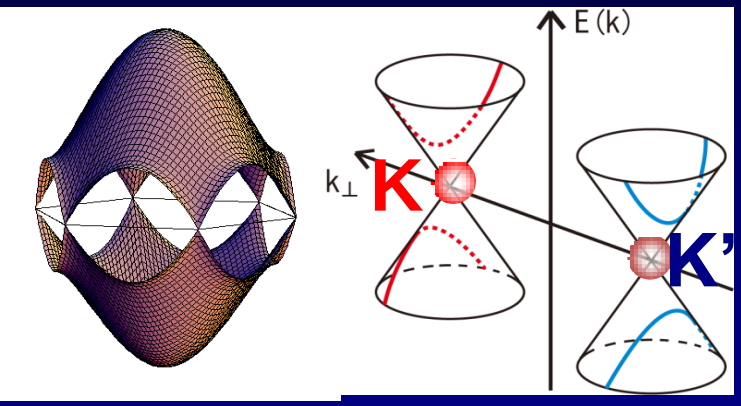
(Shima & Aoki, PRL 1993)

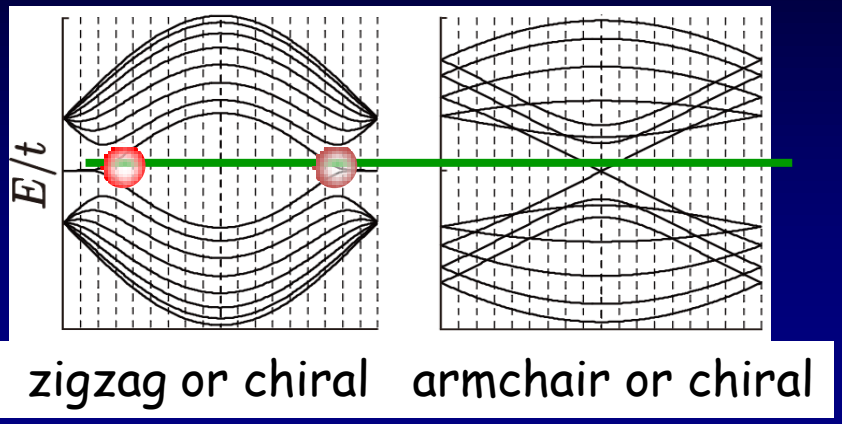


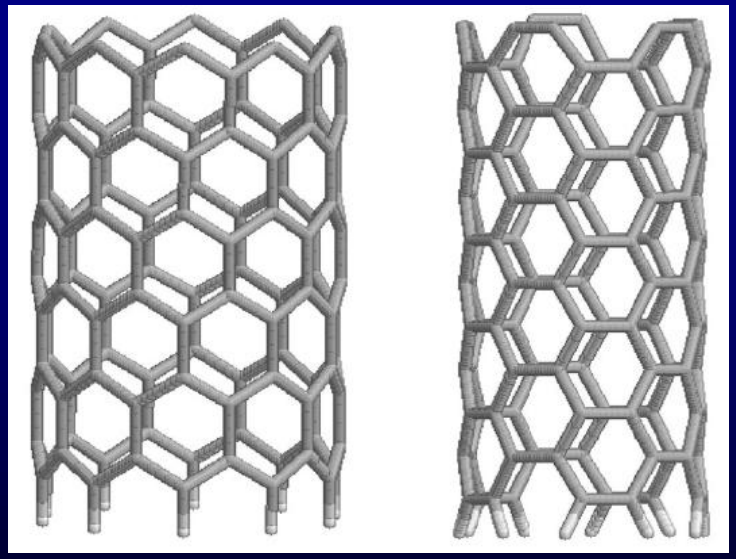


Graphene nanomesh → gap → diodes
(Bai et al, Nature Nanotech 2010)

Carbon nanotubes

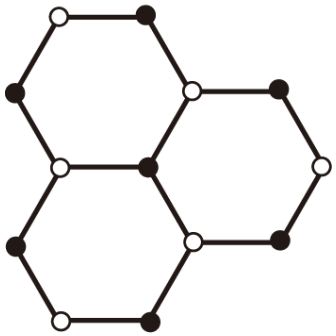






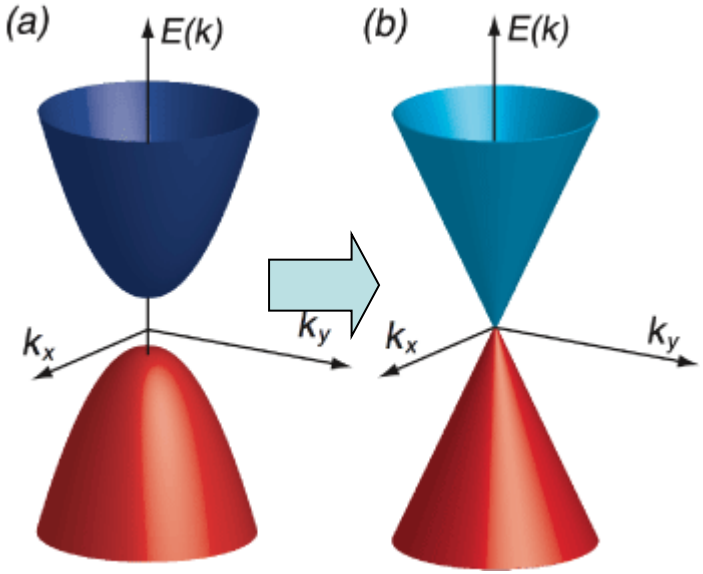
様々な系とchiral対称性

When bipartite (chirality) trivially degraded



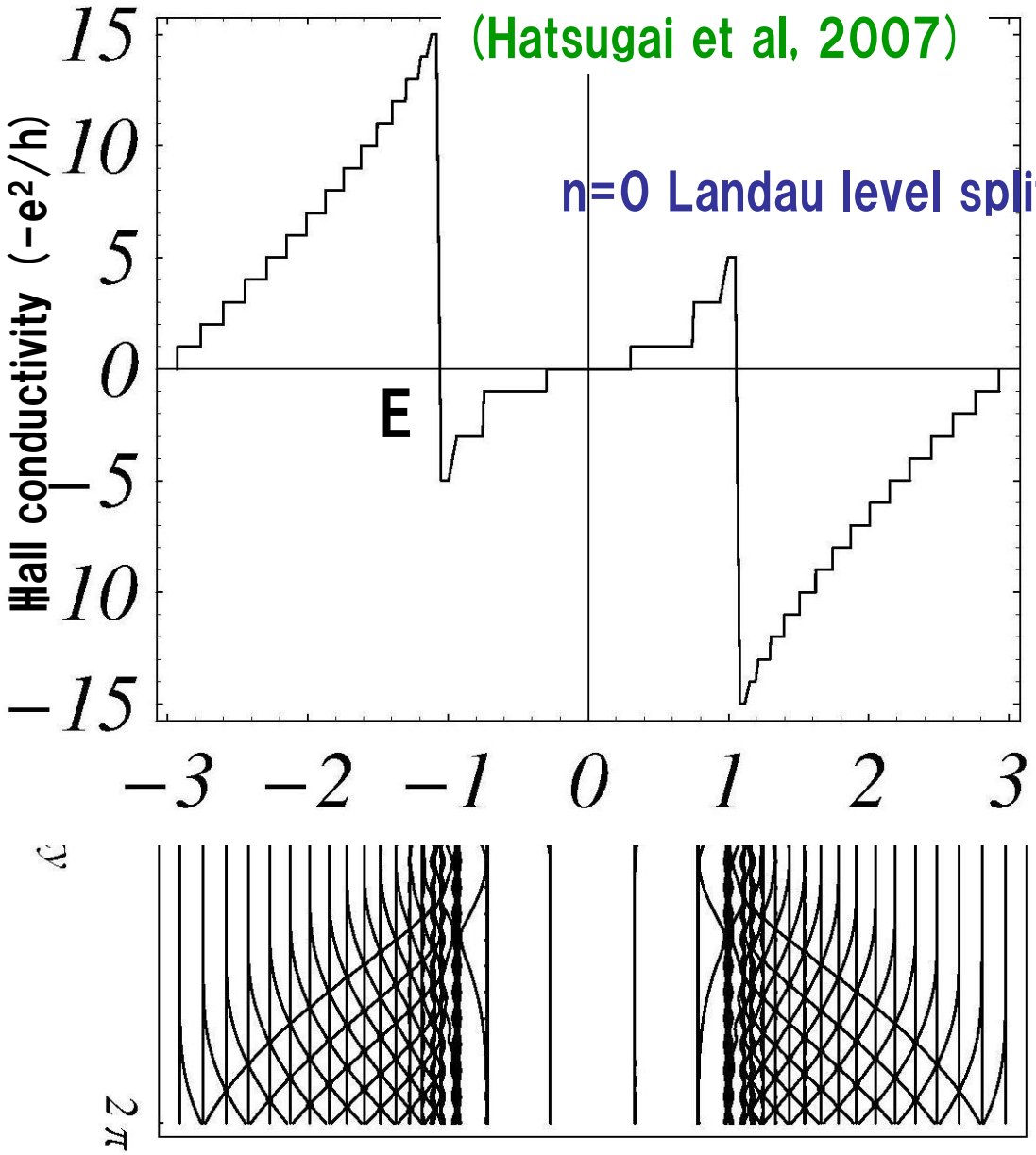
$\varepsilon_A \neq \varepsilon_B$

as in BN



(Hatsugai et al, 2007)

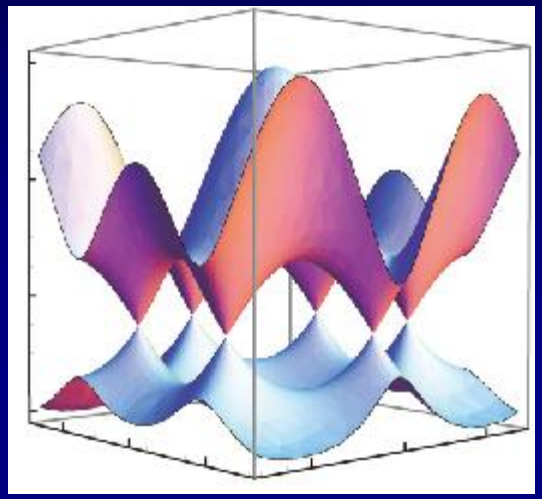
n=0 Landau level splits

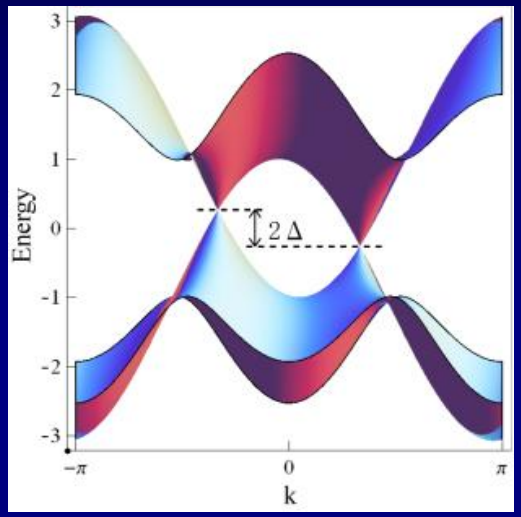


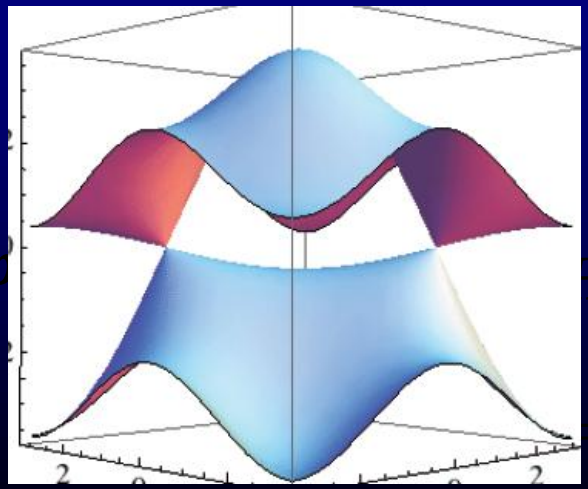
青木 秀夫: グラフェンの物理

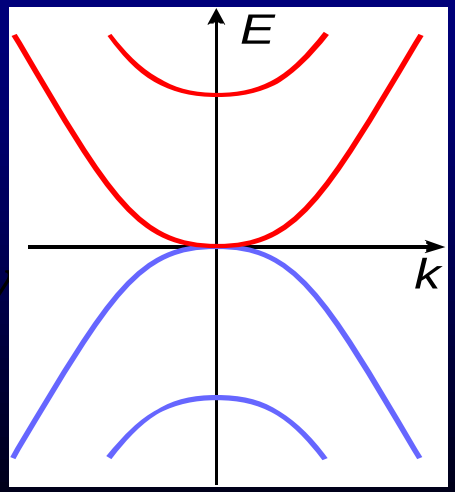
E

Various extensions
(shifted cones, tilted cones, bilayer, …)









$H_K = v_F(\sigma_x p$

$= v_F$

$H_{K'} = v$

$= v_F$

An example: graphene in B with random bonds

preserves the chiral symmetry

$$H = \sum_{<r,r'>} t_{r,r'} e^{i\theta_{rr'}} c_r^\dagger c_{r'}$$

$$t_{r,r'} = t + \delta t_{r,r'}$$

written as $\cos(\theta_i - \theta_j) = \mathbf{N}_i \cdot \mathbf{N}_j$, where $\mathbf{N}_i$ is the unit vector perpendicular to the surface, defined in Eq. (125). If $\mathbf{r}_j = \mathbf{r}_i + \mathbf{u}_{ij}$, we can write

$$\mathbf{N}_i \cdot \mathbf{N}_j \approx 1 + \mathbf{N}_i \cdot [(\mathbf{u}_{ij} \cdot \nabla)\mathbf{N}_i] + \tfrac{1}{2}\mathbf{N}_i \cdot [(\mathbf{u}_{ij} \cdot \nabla)^2 \mathbf{N}_i], \quad (162)$$

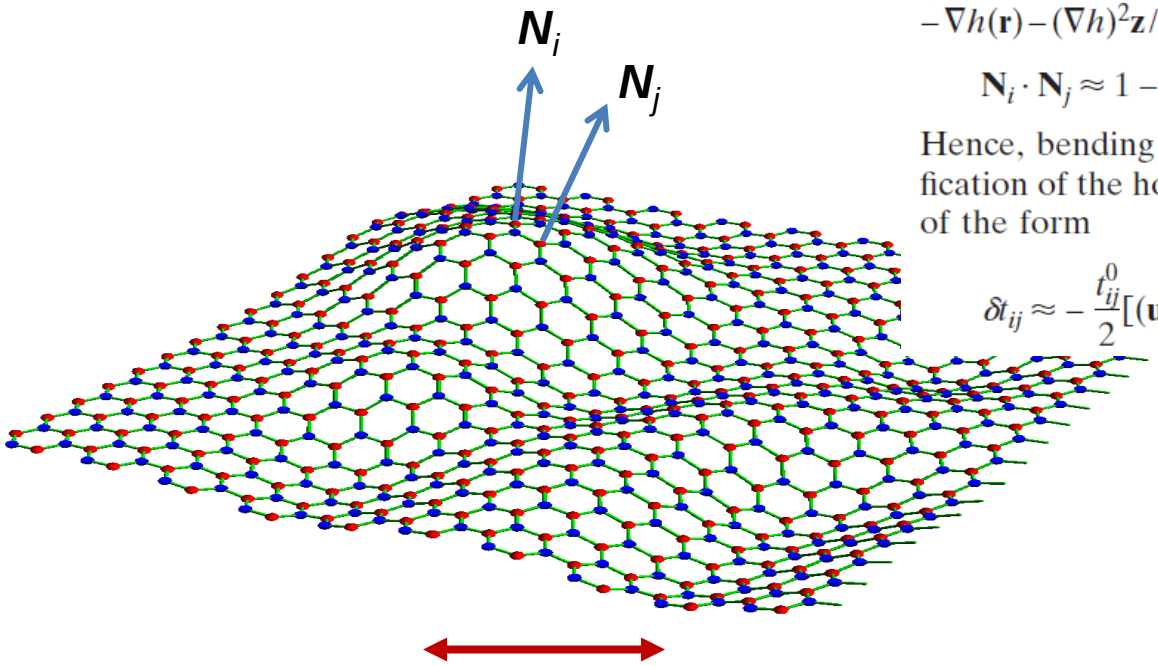
where we assume smoothly varying $\mathbf{N}(\mathbf{r})$. We use Eq. (125) in terms of the height field $h(\mathbf{r})$ $[\mathbf{N}(\mathbf{r}) \approx \mathbf{z} - \nabla h(\mathbf{r}) - (\nabla h)^2 \mathbf{z}/2]$ to rewrite Eq. (162) as

$$\mathbf{N}_i \cdot \mathbf{N}_j \approx 1 - \tfrac{1}{2}[(\mathbf{u}_{ij} \cdot \nabla)\, \nabla\, h(\mathbf{r}_i)]^2. \quad (163)$$

Hence, bending of the graphene sheet leads to a modification of the hopping amplitude between different sites of the form

$$\delta t_{ij} \approx -\frac{t_{ij}^0}{2}[(\mathbf{u}_{ij} \cdot \nabla)\, \nabla\, h(\mathbf{r}_i)]^2, \quad (164)$$

Castro Neto et al, 2009

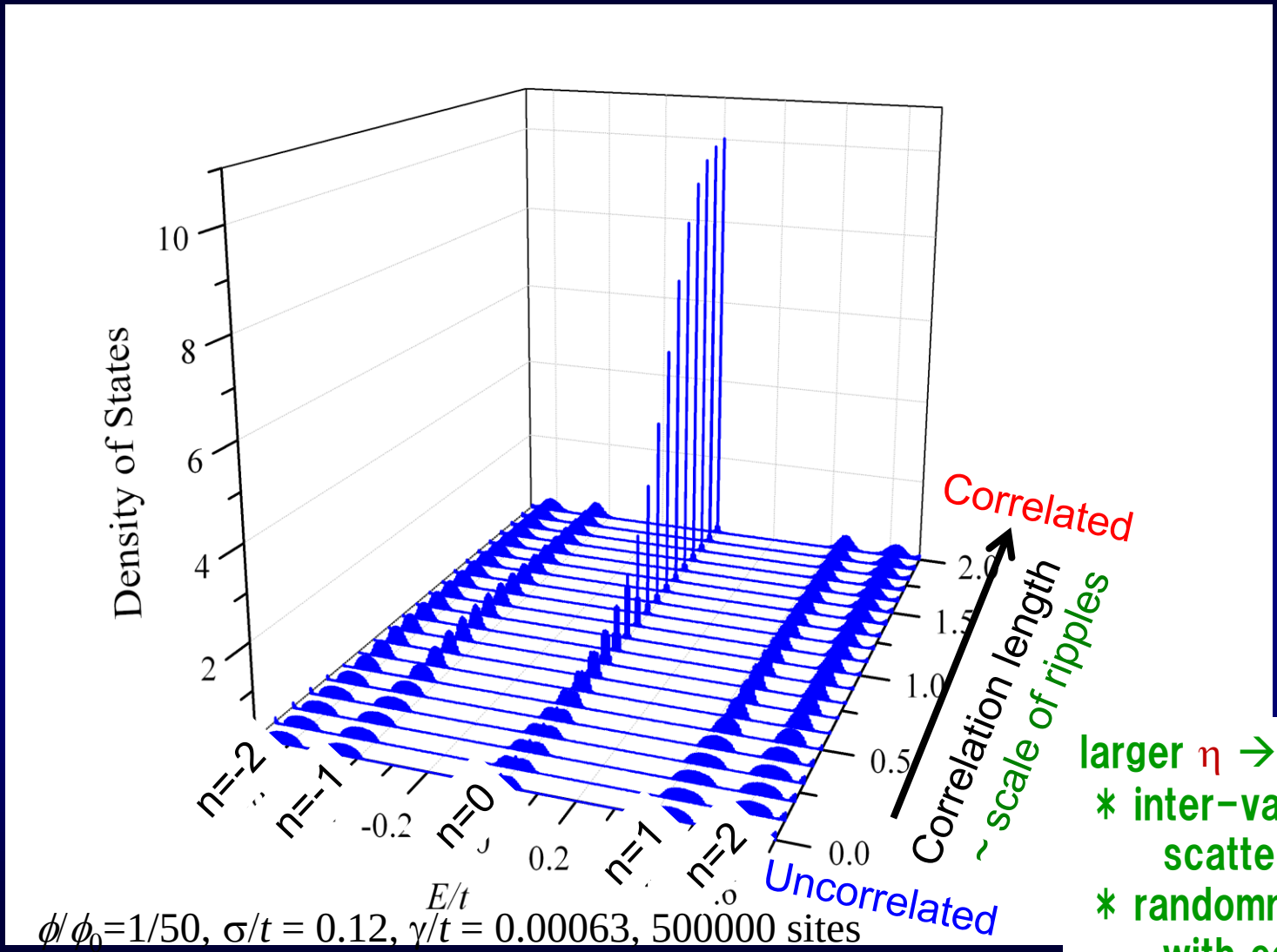


Correlated hopping:

spatial correlation length η → controlls the K-K' scattering

n=0 Landau level for bond disorder

(Kawarabayashi et al, PRL 2009)

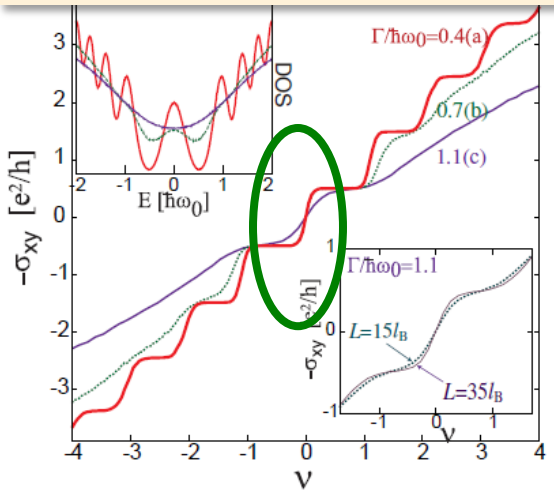


ϕ/ϕ_0=1/50, σ/t = 0.12, γ/t = 0.00063, 500000 sites

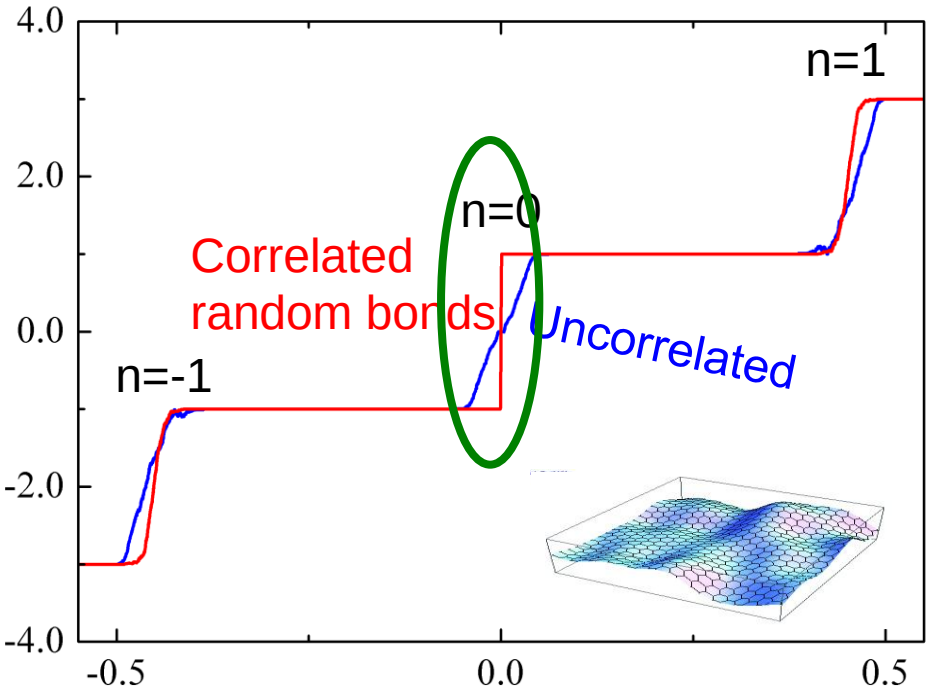
larger η →

* inter-valley (K-K') scattering suppressed
* randomness describable with continuum theory

Dirac field + Potential disorder
(Nomura *et al*, PRL 2008)



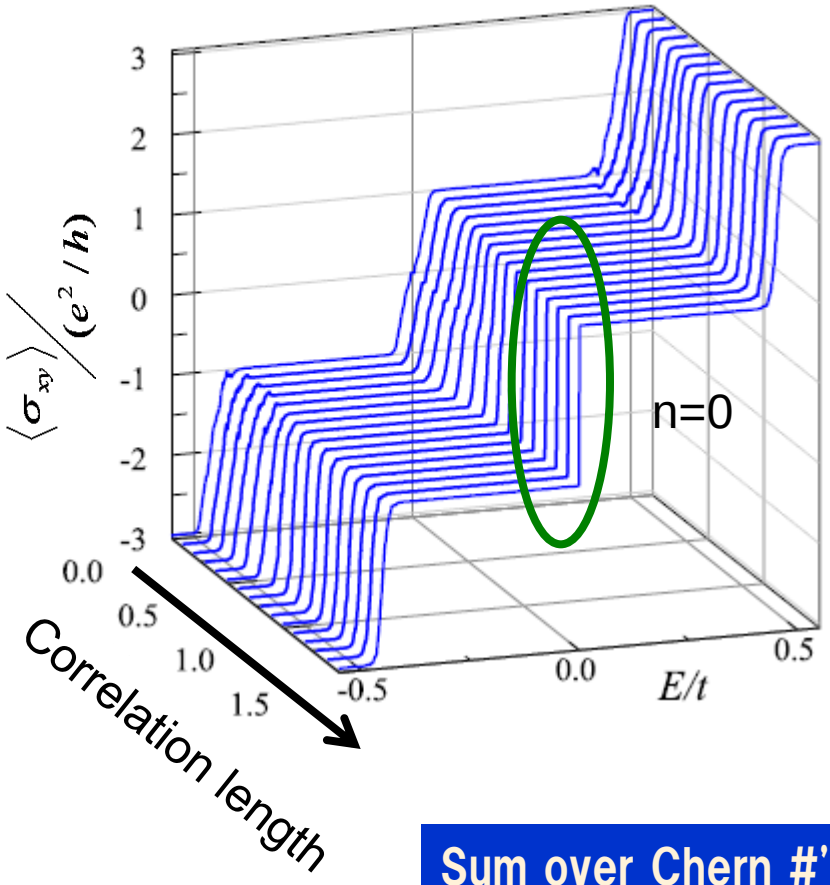
Honeycomb lattice + Bond disorder
(Kawarabayashi, Hatsugai & Aoki, PRL 2009)

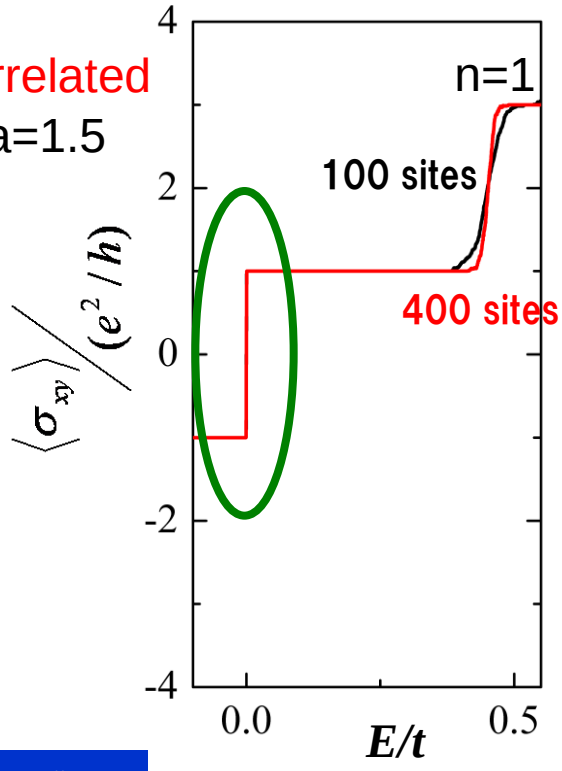


ϕ/ϕ_0=1/50, σ/t = 0.12, γ/t = 0.0

Fixed-point-like behaviour for σ_{xy}

(Kawarabayashi, Hatsugai & Aoki, PRL 2009)





Sum over Chern #'s in the "Dirac sea":
lattice-gauge technique

(an extension of the King-Smith-Vanderbilt 1st principles formalism to Chern numbers)

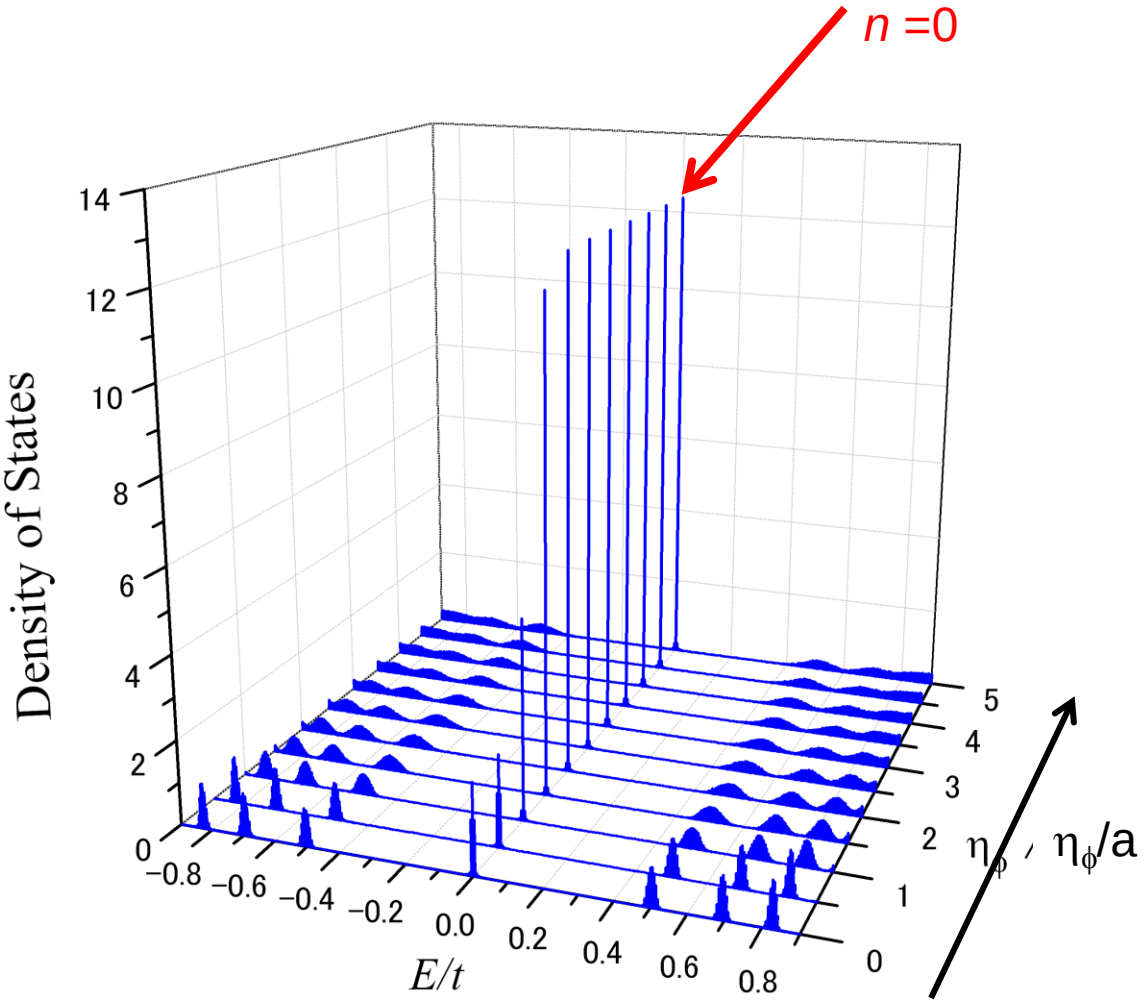
Random magnetic fields (chiral symmetry preserved)

$\phi+\delta\phi(r)$

$\phi/\phi_0=1/41$

$\sigma_\phi/\phi = 0.24$

(Kawarabayashi, Hatsugai & Aoki)



Chiral Symmetry
(Particle-Hole Symmetry)



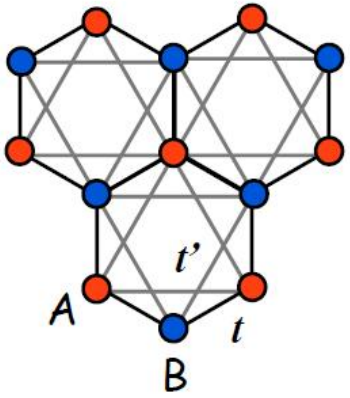
?

Anomaly (no broadening)

Additional Next Nearest-Neighbor hopping t'

Graphene: $t'/t = 0.2 \sim 0.02$

S. Reich et al. PRB66, 035412 (2002)
Nakajima, Aoki Physica E (2008)



~~Bipartite~~

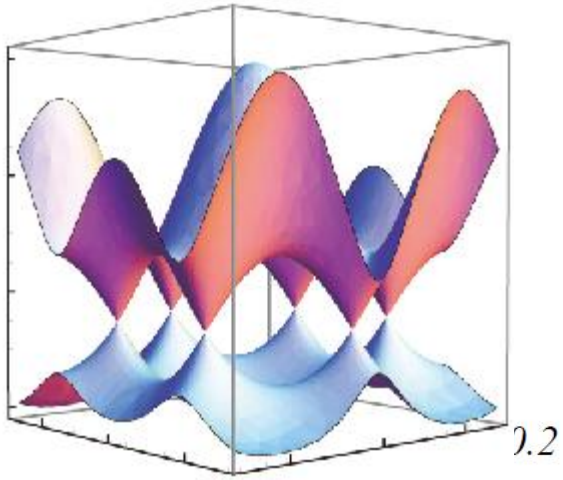
~~Chiral symmetry~~

~~Particle-Hole symmetry~~

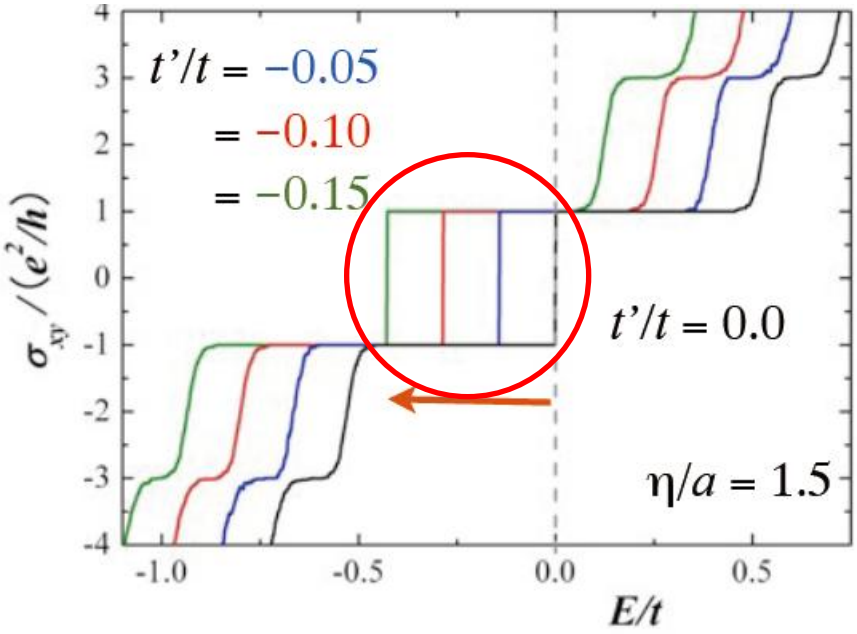
Will the anomalous QHE be washed out ?

Dirac cones at K and K'

$$E(\boldsymbol{q}) \sim -3t' \pm v_F|\boldsymbol{q}|$$
$$\boldsymbol{q} = \boldsymbol{k} - \boldsymbol{K}, \boldsymbol{k} - \boldsymbol{K}'$$



(Kawarabayashi et al, PRB 2010)

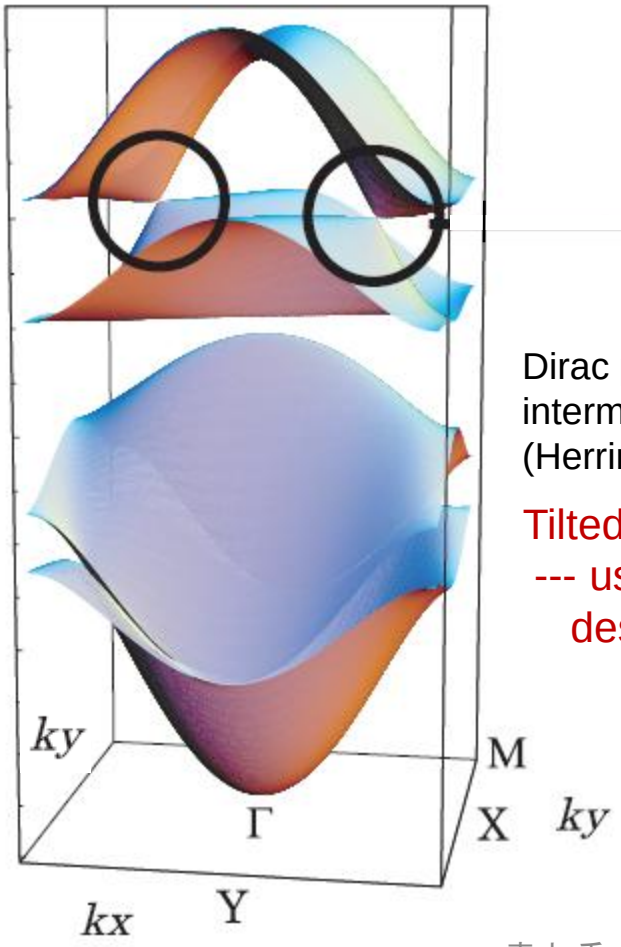


Anomaly arises as far as Dirac cones exist as the low-E effective theory

"Tilted" Dirac cones in organics

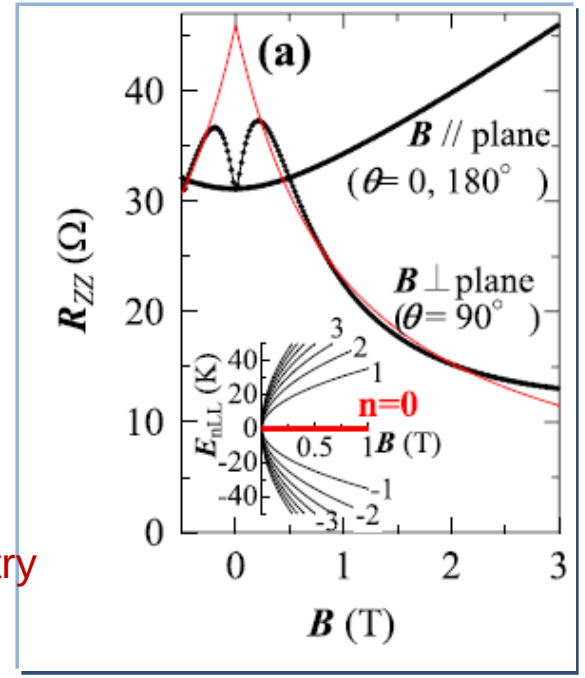
α-$(BEDT\text{-}TTF)_2I_3$

(Tajima et al, JPSJ 2000; 2002; 2006; Kobayashi et al, JPSJ 2004;
片山et al, 日本物理学会誌 62, 99 (2007))



Dirac point at an intermediate position in BZ (Herring)

Tilted Dirac cones
--- usual chiral symmetry destroyed



(Tajima et al, PRL 2009)

What are the consequences of tilting ?

Vertical Dirac cone



Graphene

K.S. Novoselov et al.,
Nature 438, 197 (2005)
...

Tilted Dirac cone



α-(BEDT-TTF)$_2$I$_3$

N. Tajima et al., PRL102, 176403 (2009)
M.O. Goerbig et al., PRB 78, 045415 (2008)
A. Kobayashi et al., JPSJ 76, 034711 (2007)

Landau levels

$$E_n = \pm\sqrt{2eB\hbar v_F^2|n|}$$

Only a Modification of Factor

$$E_n = \pm\lambda\sqrt{2eB\hbar v_F^2|n|}$$

T. Morinari et al., JPSJ 78, 023704 (2009)

Chiral symmetry

$$\sigma_z H \sigma_z = -H$$

No chiral symmetry

$$\gamma H \gamma = -H$$

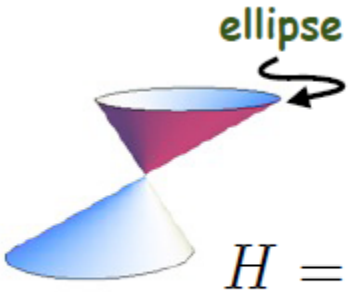
Robustness of zero modes
(n=0 Landau level)

Any other symmetry ?

Robustness of zero modes ?

"Generalised chirality" for tilted Dirac cones

(Kawarabayashi et al, PRB 2011)



$$H = \sigma_0(\boldsymbol{W}\cdot\boldsymbol{\pi}/\hbar) + (\boldsymbol{\sigma}\cdot\boldsymbol{X})\pi_x/\hbar + (\boldsymbol{\sigma}\cdot\boldsymbol{Y})\pi_y/\hbar$$

$\boldsymbol{\pi} = \boldsymbol{p} - e\boldsymbol{A}$

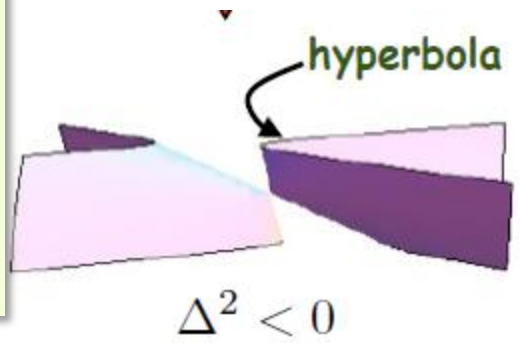
$\Delta^2 > 0$

We can still define $\gamma = \boldsymbol{\sigma}\cdot[(\boldsymbol{X}\times\boldsymbol{Y}) - \mathrm{i}(W_y\boldsymbol{X} - W_x\boldsymbol{Y})]/\Delta$

$$\gamma^\dagger H\gamma = -H, \qquad \Delta^2 = |\boldsymbol{X}\times\boldsymbol{Y}|^2 - (W_y\boldsymbol{X} - W_x\boldsymbol{Y})^2$$

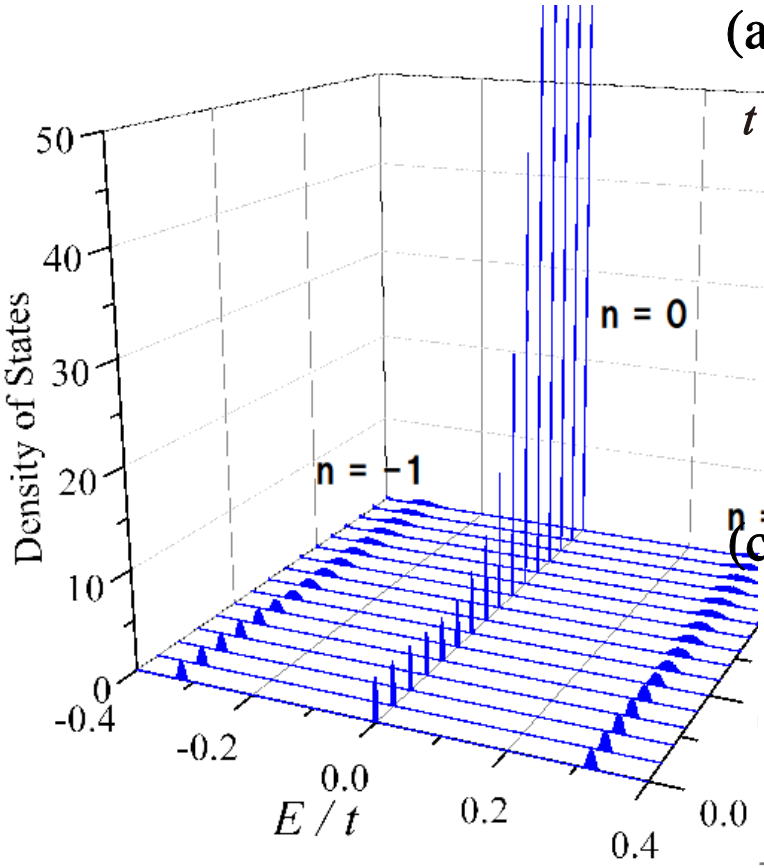
γ: nonhermitian extension of chiral operator, with eigenvalues ± 1, s $\gamma^2 = (\gamma^\dagger)^2 = \sigma_0$

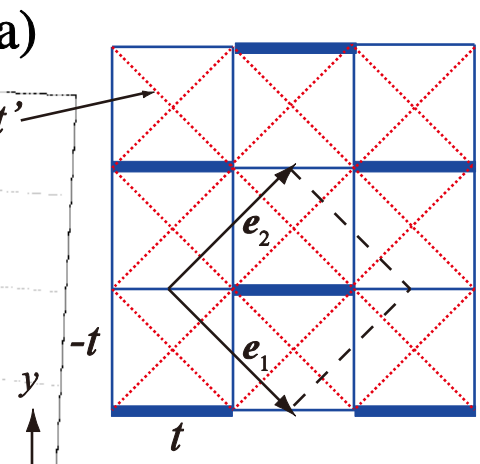
Generalised chiral symm definable
←→ (elliptic) Dirac cone dispersion
←→ *H*: elliptic as a differential operator
←→ index theorem applicable
--- rigorous link!

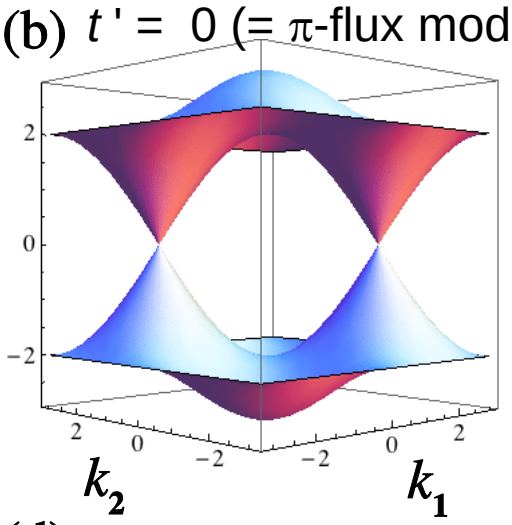


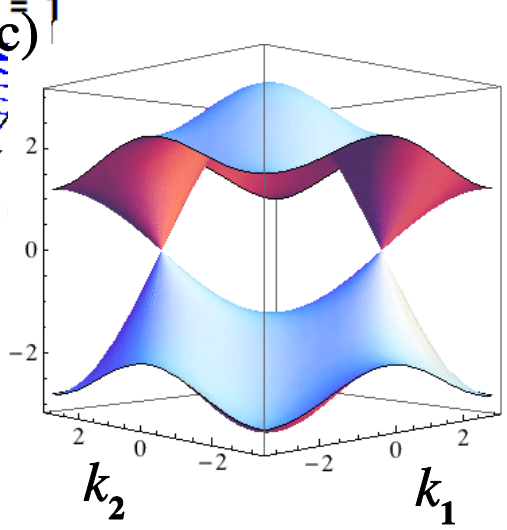
Gen. chiral symm → δ-function-like n=0 LL analytically (Aharonov-Casher argument)

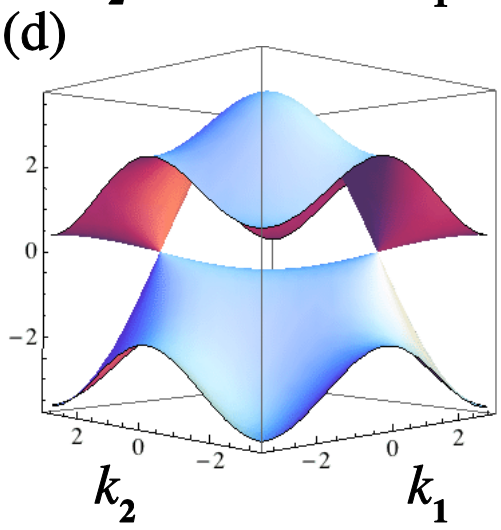
(Kawarabayashi et al, arXiv:1101.4273)











Aharonov-Casher argument

Degeneracy of zero-modes in the usual case (Aharonov-Casher 1979)
→ extended to the generalised chiral symm
(Kawarabayashi et al, arXiv:1101.4273)

Dirac-Landau eqn:

$$\begin{bmatrix} 0 & \boldsymbol{\alpha}\cdot\boldsymbol{\pi} \\ \boldsymbol{\alpha}^*\cdot\boldsymbol{\pi} & 0 \end{bmatrix} \begin{bmatrix} \psi_+ \\ \psi_- \end{bmatrix} = E \begin{bmatrix} 1 & \beta \\ \beta^* & 1 \end{bmatrix} \begin{bmatrix} \psi_+ \\ \psi_- \end{bmatrix}$$

→ $\mathcal{D}_\pm \psi_\pm = (\partial_{\tilde{R}_x} \pm \mathrm{i}\chi\partial_{\tilde{R}_y})\psi_\pm = 0$

R : "principal-axis" coord λ : "ellipticity"

$(\tilde{R}_x, \tilde{R}_y) = (\lambda^{-1}R_x, \lambda R_y)$ $\chi = \mathrm{sgn}(\mathrm{Im}(\alpha_X \alpha_Y^*))$

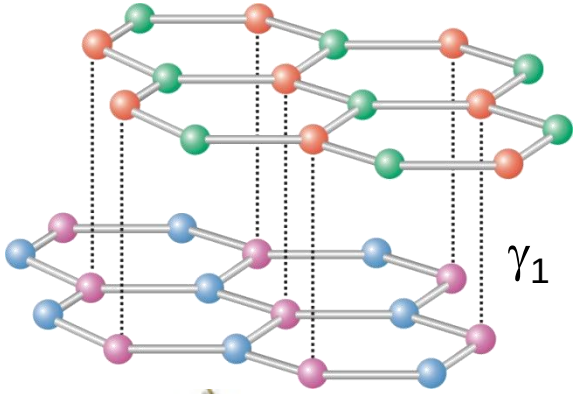
→ $\tilde{\psi}_\pm$ is an *entire function* of $Z_\pm = \tilde{R}_x \pm \mathrm{i}\chi\tilde{R}_y$

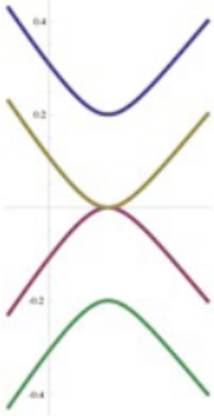
→ degeneracy of zero modes is Φ/ϕ_0

= degeneracy of LL

→ *no broadening occurs for the* $n = 0$ *Landau level.*

Bilayer graphene





Zero Gap

Quadratic dispersion

Landau Levels

$$\hat{H} = -\frac{\hbar^2}{2m}\begin{pmatrix} 0 & (k_x - ik_y)^2 \\ (k_x + ik_y)^2 & 0 \end{pmatrix}$$

$$E_n = \pm\hbar\omega_c\sqrt{n(n-1)}$$

McCann and Fal'ko, PRL 96, 086805 (2006)

Zero mode : n=0 and n=1

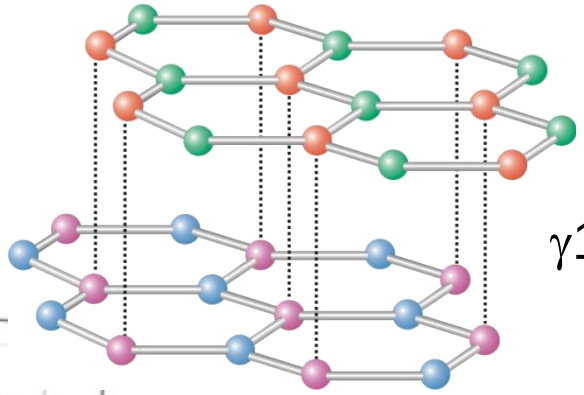
Katsnelson, Prokhorova, PRB 77 (2008)

Graphite ～ single-layer graphen + bilayer graphene
(Koshino & Ando, PRB 2007)

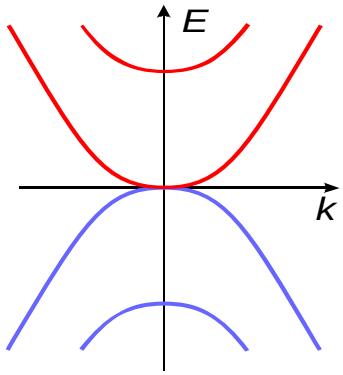
Bilayer graphene (Kawarabayashi et al, in prep)

Chiral symmetric model

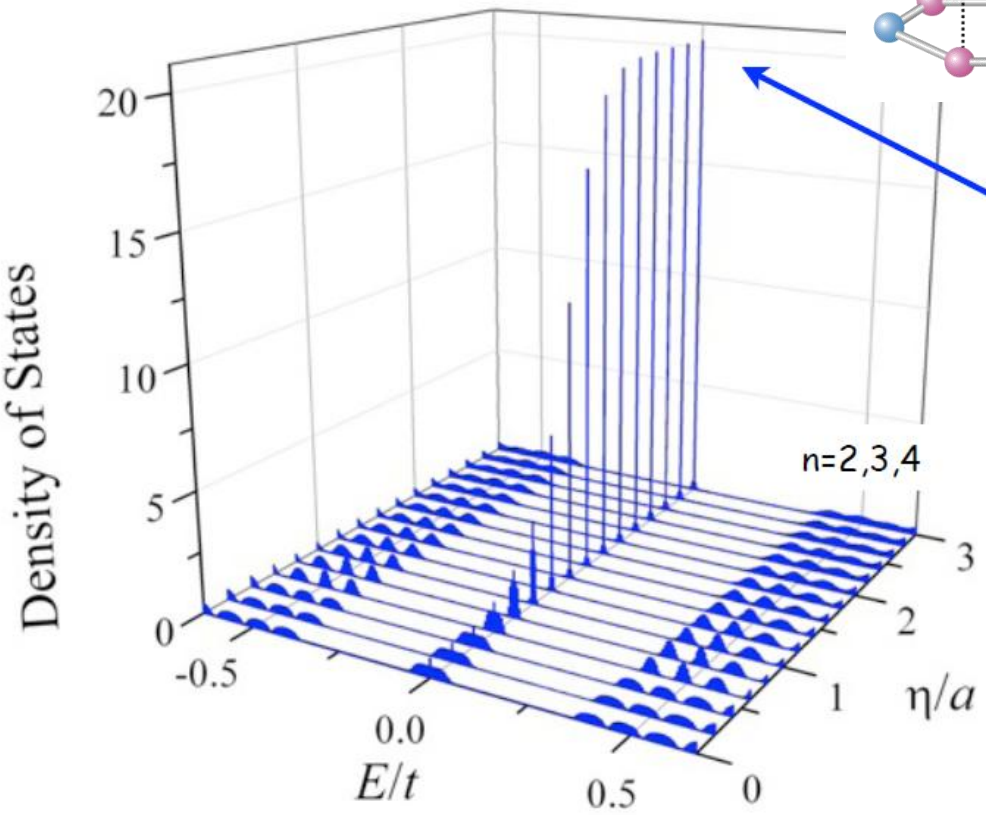
No Dirac cones



γ1



n=0 and n=1

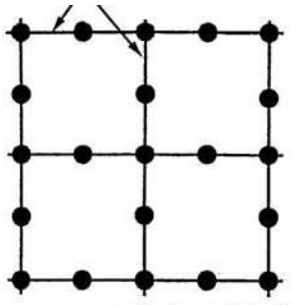


Zero mode anomaly
for correlated bond disorder

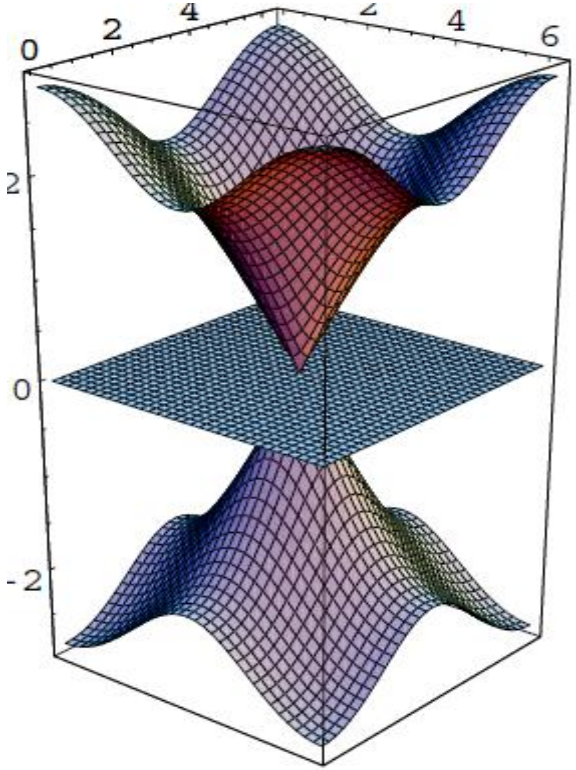
fourfold degenerated (per spin)

Chiral symmetry is relevant

Can we go around Nielsen-Ninomiya ?



Flat-band (dp)model
(Aoki et al, 1996)

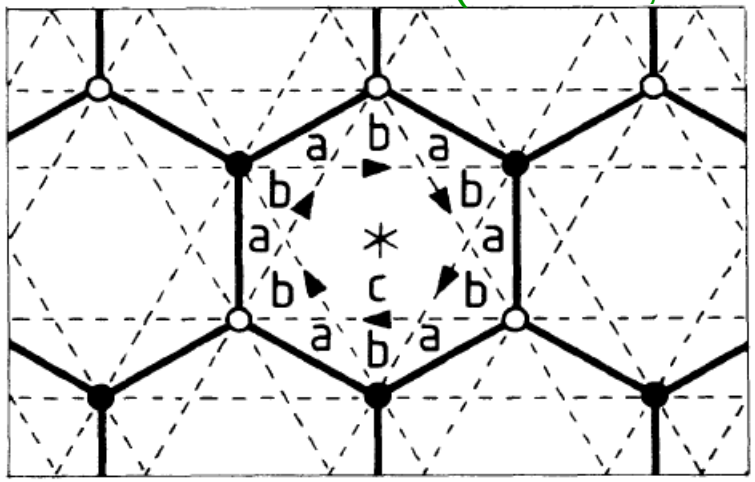


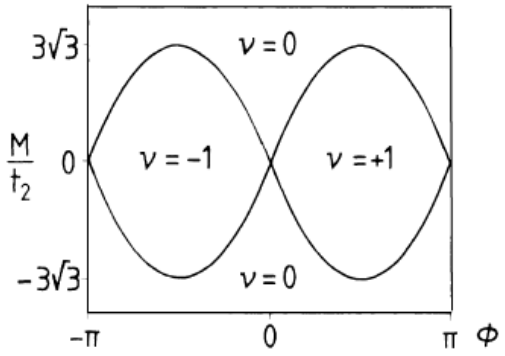
B

Can we manipulate two Dirac cones ?

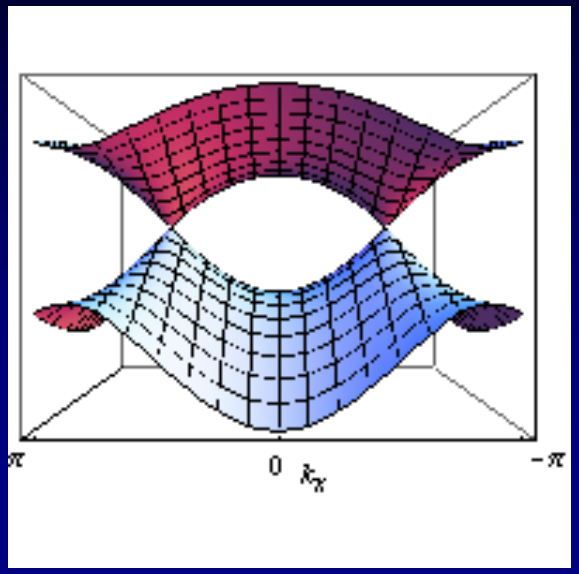
QHE without Landau levels

(Haldane, 1988)

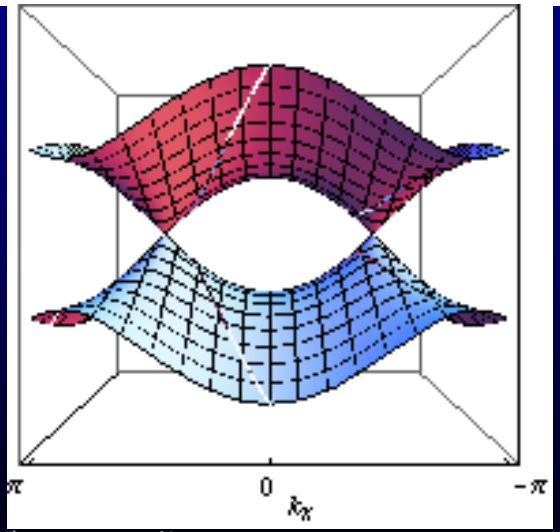




ν: Chern #, M = $\varepsilon_A - \varepsilon_B$



(Watanabe et al, PRB(R) 2010)



- 場の理論では、厳密に

$$\sigma_{xy} = \frac{e^2}{2h}\mathrm{sgn}(m)$$

(半整数)であることが示せる:

$$\mathcal{H}(\boldsymbol{k}) = k_1\sigma_1 + k_2\sigma_2 + m\sigma_3$$

$$\mathcal{A}_n(\boldsymbol{k}) = i\langle n(\boldsymbol{k})|\boldsymbol{\nabla}|n(\boldsymbol{k})\rangle$$

$$\mathcal{F}_n(\boldsymbol{k}) = [\boldsymbol{\nabla}\times\mathcal{A}_n(\boldsymbol{k})]_3$$

$$\sigma_{xy} = -\frac{e^2}{h}\int_{R^2}\frac{d^2\boldsymbol{k}}{2\pi}\mathcal{F}_n(\boldsymbol{k})$$

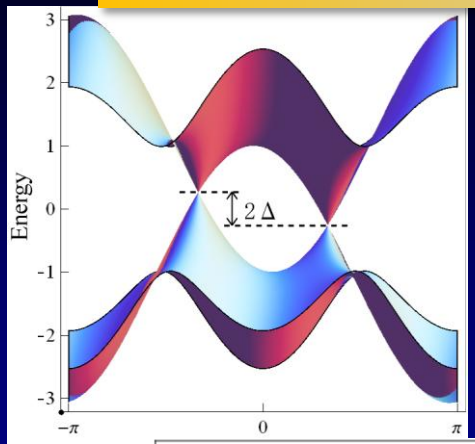
(Niemi et al, PRL 1983;
Redlich, PRL 1984)

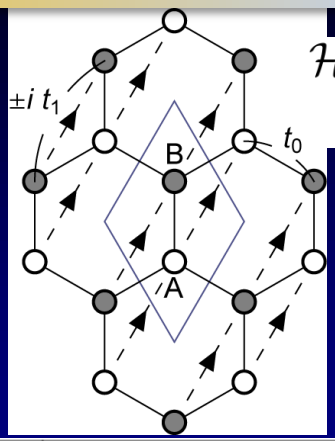
- Lattice modelでは必ず integer。
(Thouless et al, PRL 1982)

cf. Nielsen-Ninomiyaの定理
(Nielsen & Ninomiya, Nucl. Phys. B 1981)

Shifted Dirac cones

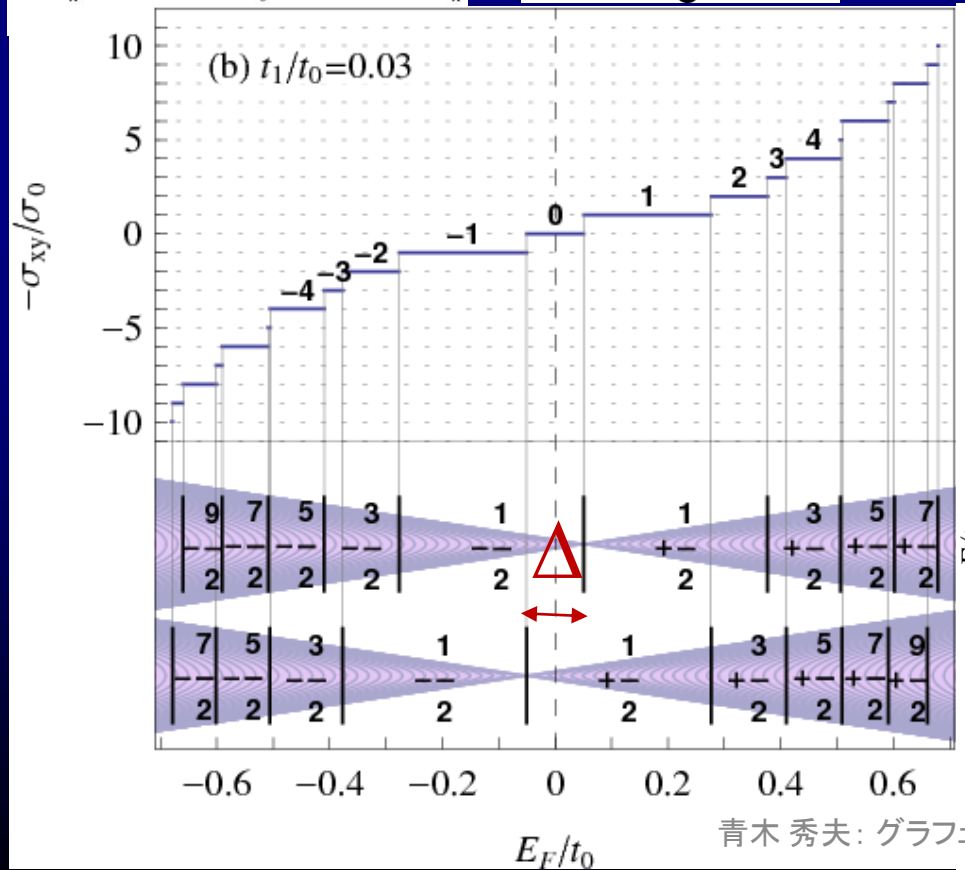
(Watanabe et al, PRB(R) 2010)

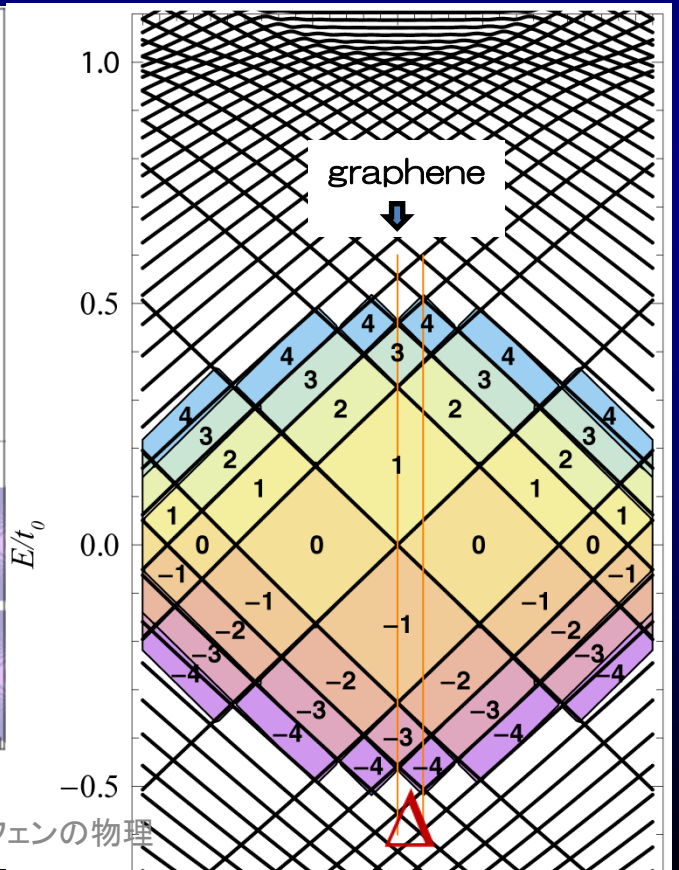




$$\mathcal{H} = \chi\Delta\sigma_0 - hv_F[\chi\delta k_x\sigma_1 + \delta k_y\sigma_2]$$

χ = 1 (-1) for K(K')





青木 秀夫: グラフェンの物理

Complex hopping realisable in cold atoms in optical lattices

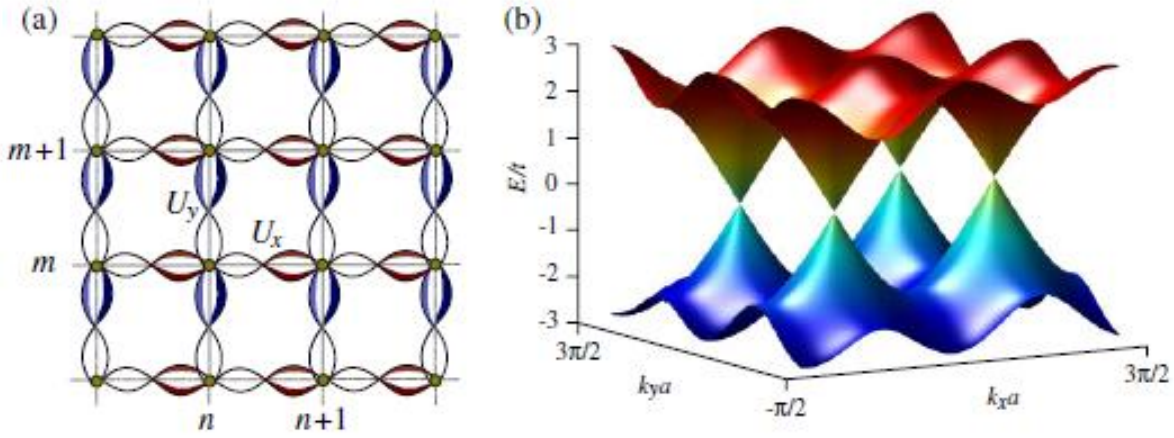
(Bermudez et al, PRL 2009)

$$H = -t \sum \sum c_{\tau'}^{\dagger}(\mathbf{r}') e^{-i \int_{\mathbf{r}}^{\mathbf{r}'} \mathbf{A}\cdot\mathbf{dl}} c_{\tau}(\mathbf{r})$$

Two-component(hyperfine) fermion systems

→ $\boldsymbol{A} = \frac{B_0}{2}(-y, x) + a(B_\alpha \sigma_y, B_\beta \sigma_x)$

Abelian Nonabelian (Wilczek-Zee 84)

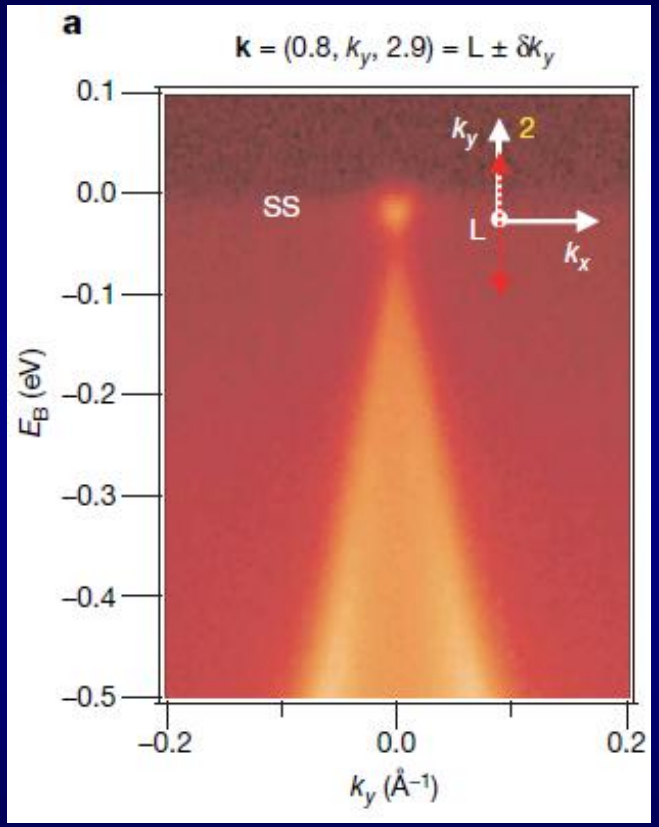


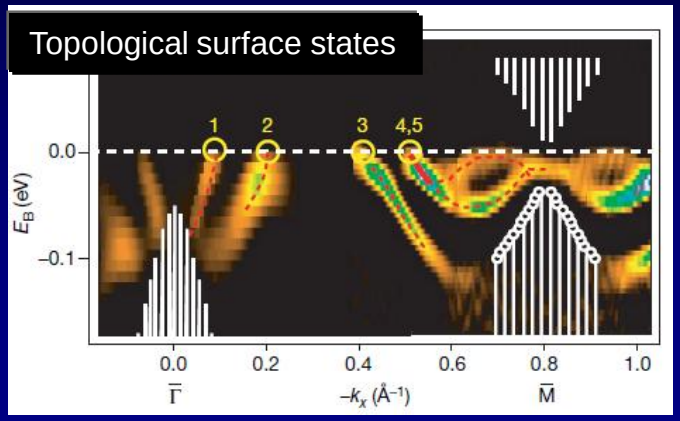
Surface of topological insulators

A topological Dirac insulator in a quantum spin Hall phase in $Bi_{1-x}Sb_x$ (Bulk: Dirac insulator; Surface: Topological metal)

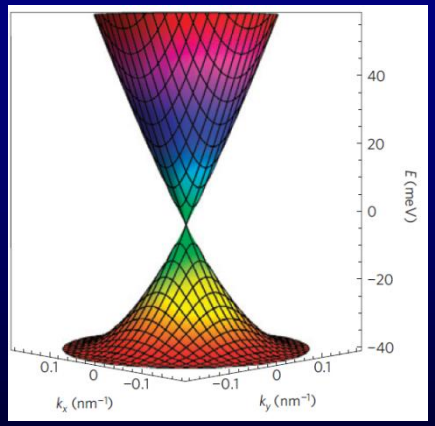
D. Hsieh[1], D. Qian[1], L. Wray[1], Y. Xia[1], Y. S. Hor[2], R. J. Cava[2] & M. Z. Hasan[1,3]

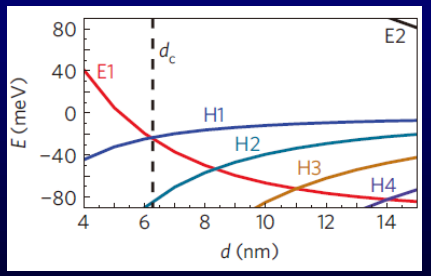
Nature 452, 970 (2008)





More recently, Buttner et al: Single Dirac fermion in zero-gap HgTe quantum wells (Nature Phys, 2011)

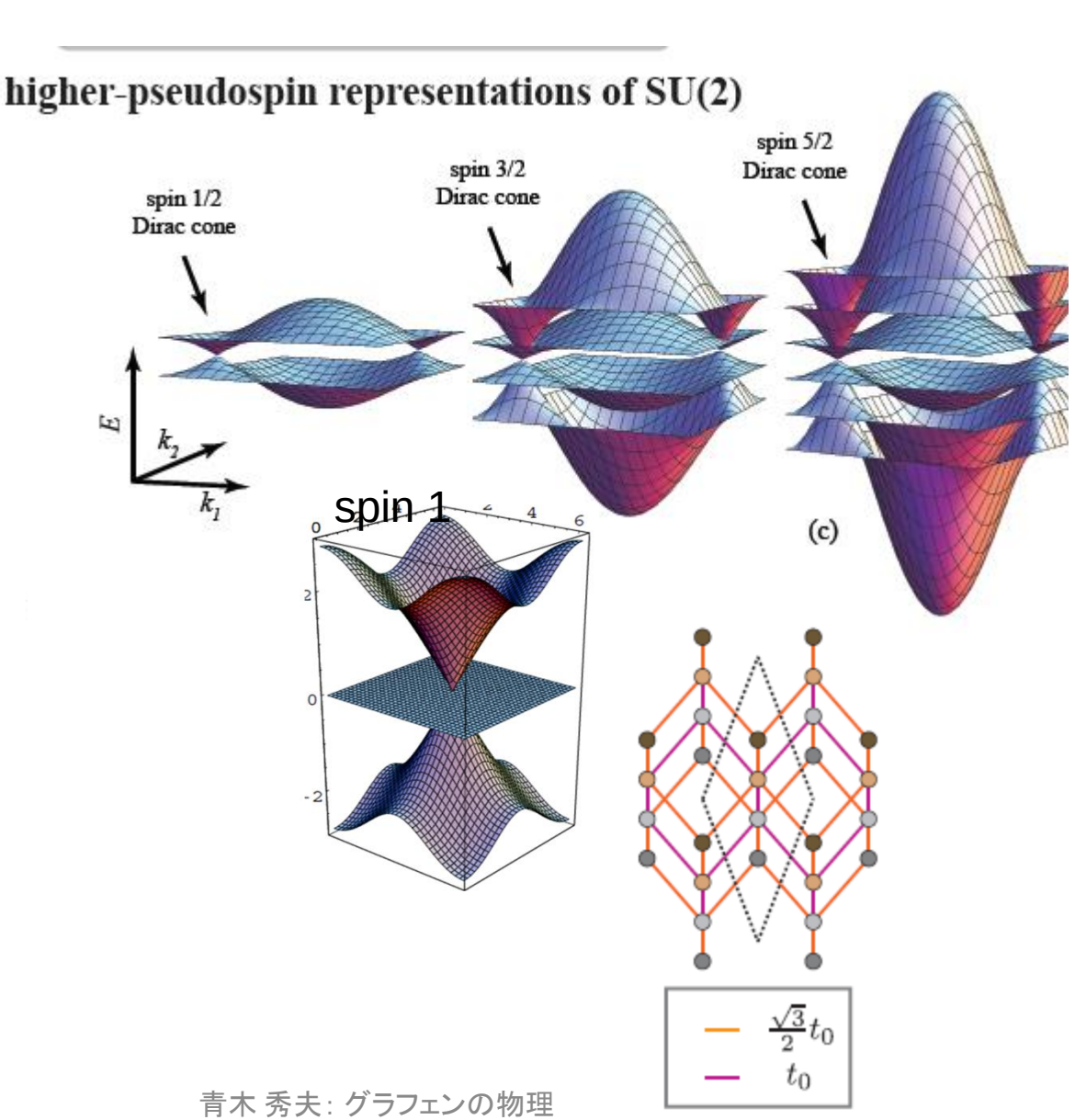


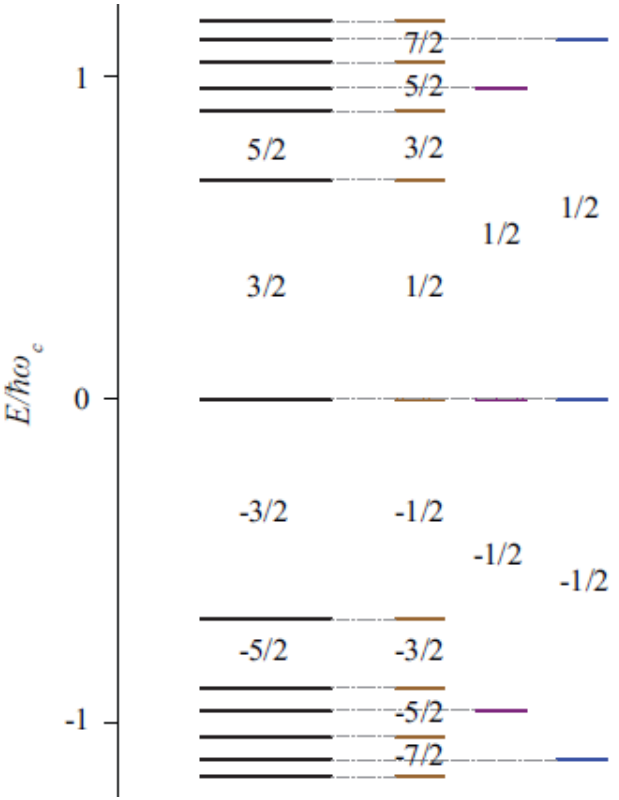


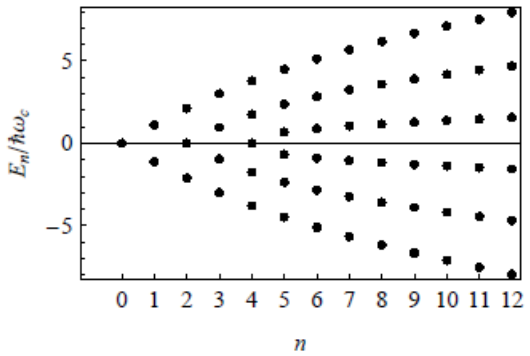
For a review on the topological insulator, see, e.g., Hasan & Kane, Rev Mod Phys **82**, 3045 (2010)

Spin S rep of SU(2) → each Dirac pt comprises (S+1/2) set(s) of cones

(Watanabe et al, arXiv:1009.1959)

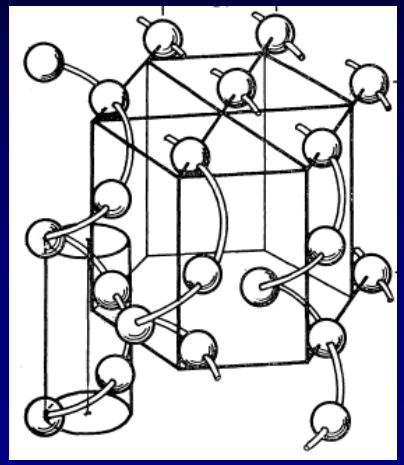




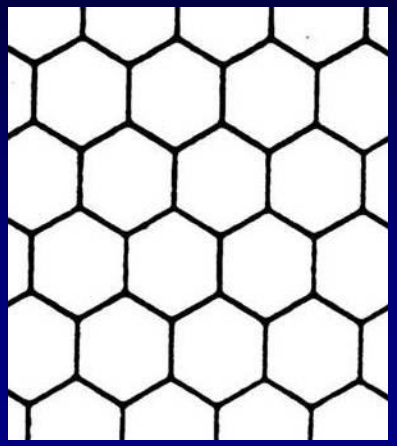


Dirac cones in spatial D ≠ 2 ?

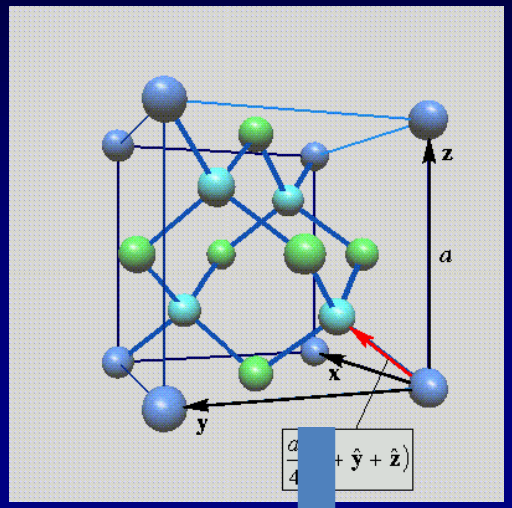
1D (Se, Te)



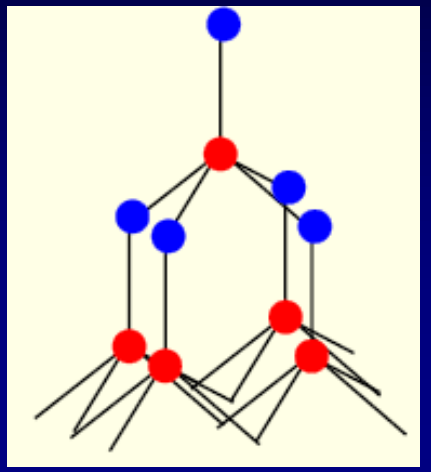
2D (graphite)



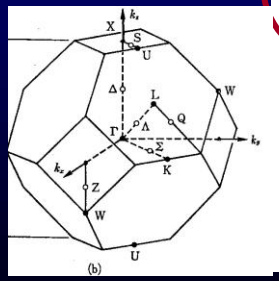
3D (diamond)

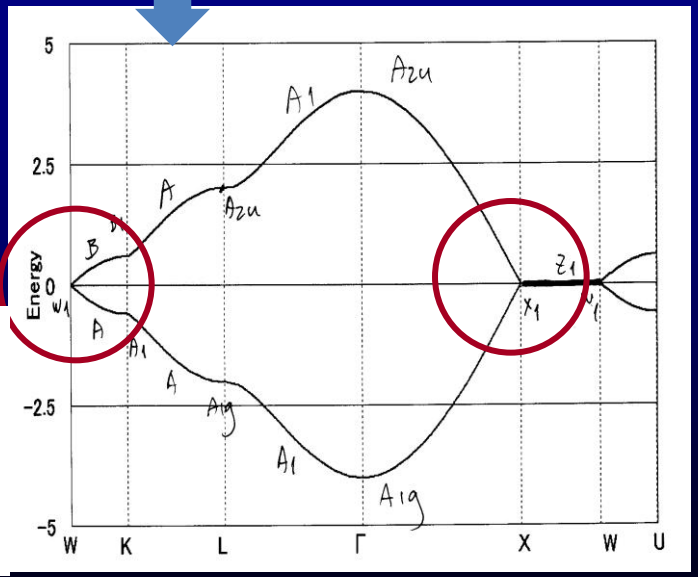


4D (Creutz, JHEP2008)



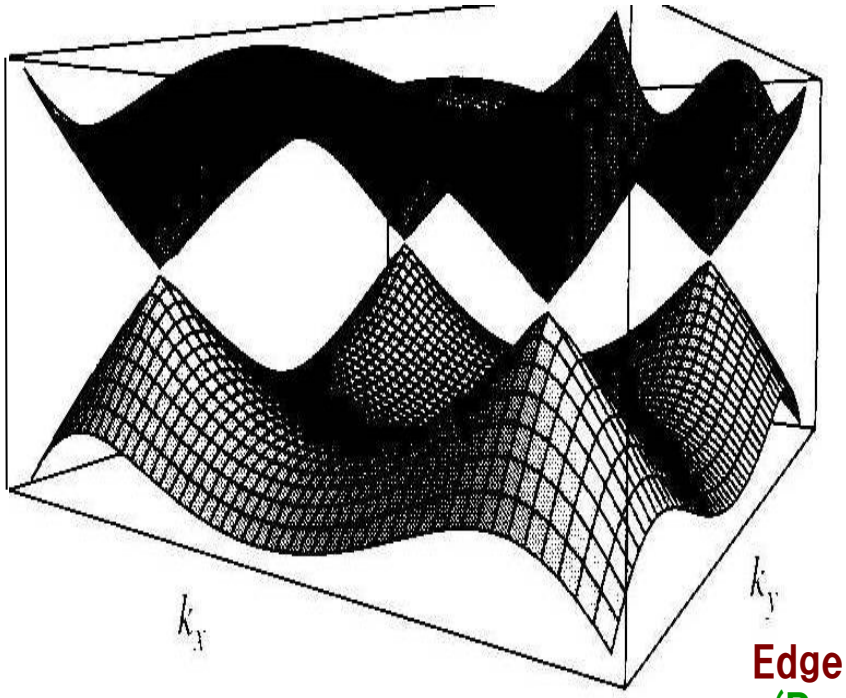
helical symmetry → band sticking
(see, eg, Heine 1960)

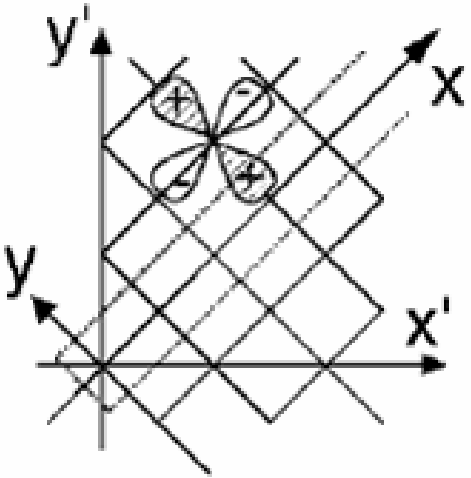




Dirac cones in d-wave superconductors

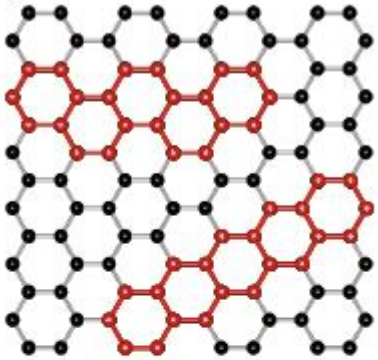
Dispersion of the quasi-particle
in d-wave superconductor with the Bogoliubov Hamiltonian
→ 2D Dirac
(See, e.g., Altland & Simons: Condensed Matter Field Theory, §6.4)



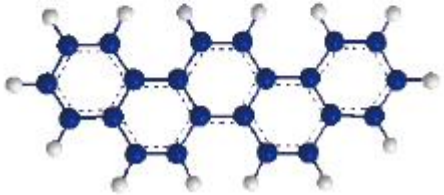


Edge states in anisotropic superconductors
(Ryu & Hatsugai, PRL 2002)

(d)



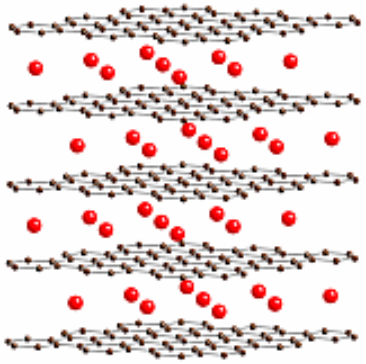
(e)



青木 秀夫: グラフェンの物理

Carbon-based superconductors

● Graphite intercalation compounds（GIC）



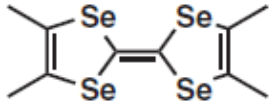
1965 AC_8 (A=K,Rb,Cs) T_c<1K **First GIC SC** Hannay *et al.*

2005 CaC_6 T_c=11.5K Emery *et al*.

● Molecular solids

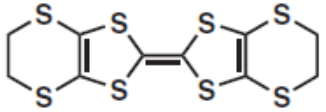
1980 $(TMTSF)_2PF_6$ 12kbar T_c=0.9K **First organic SC** Jerome *et al*.

TMTSF →



2003 β'-$(BEDT\text{-}TTF)_2ICl_2$ 82kbar T_c=14.2K Taniguchi *et al*.

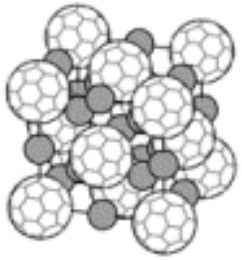
BEDT-TTF→



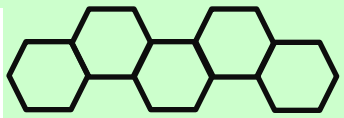
● Solid fullerene

1991 K_3C_{60} T_c=18K **First fullerene SC** Hebard *et al*.

1991 Cs_2RbC_{60} T_c=33K Tanigaki *et al*.



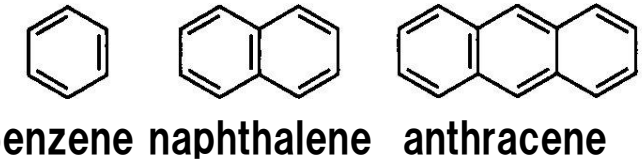
● 2009 K_x picene (x～3) T_c=7-20K Mitsuhashi *et al*. **First aromatic SC**



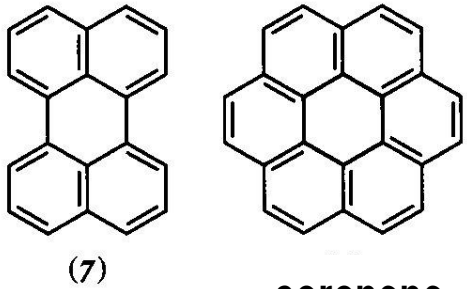
青木 秀夫: グラフェンの物理

What is an aromatic molecule ?

● aromatic molecules ＝ most basic of organic molecules
－ real surprise for its SC

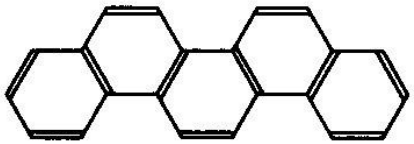


benzene naphthalene anthracene



(7) coronene

Superconductivity in coronene as well (Tc ～ 15K; Kubozono)

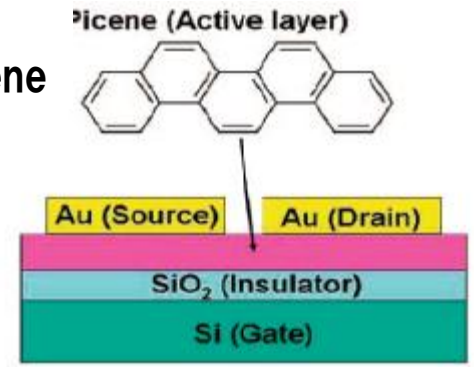
picene 

- very good insulator
- more stable than pentacene



cf. polyacethylene

FET



青木 秀夫: グラフェンの物理

room T

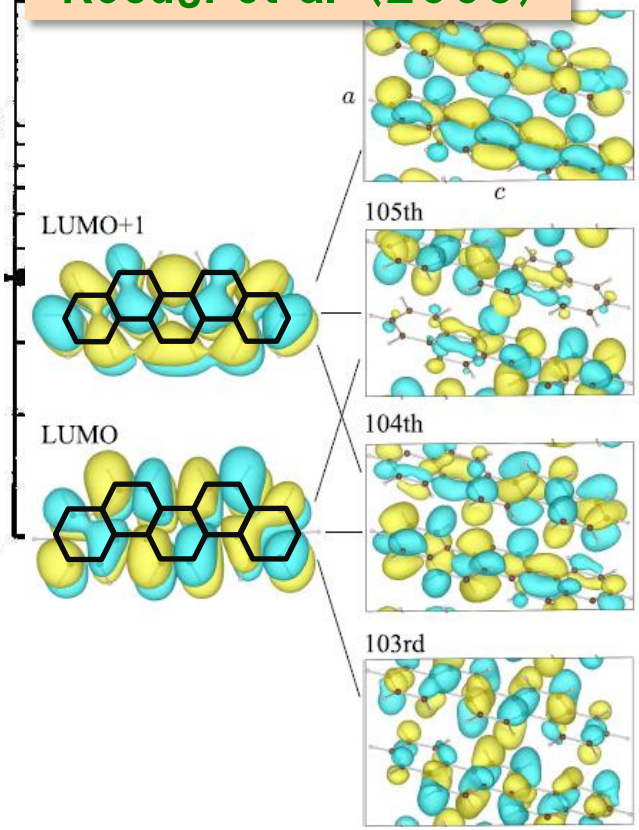
liq N

liq ^{4}He

Tc (K)



Kosugi et al (2009)



year

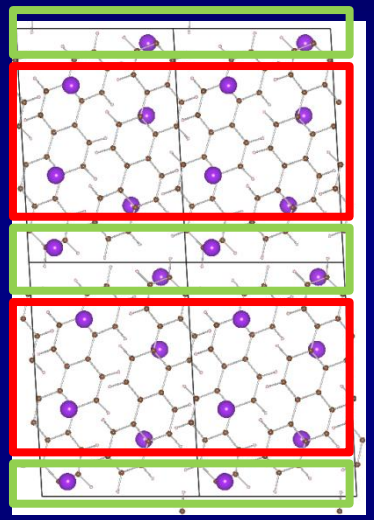
1st aromatic SC →
Mitsuhashi et al, nature (2010)

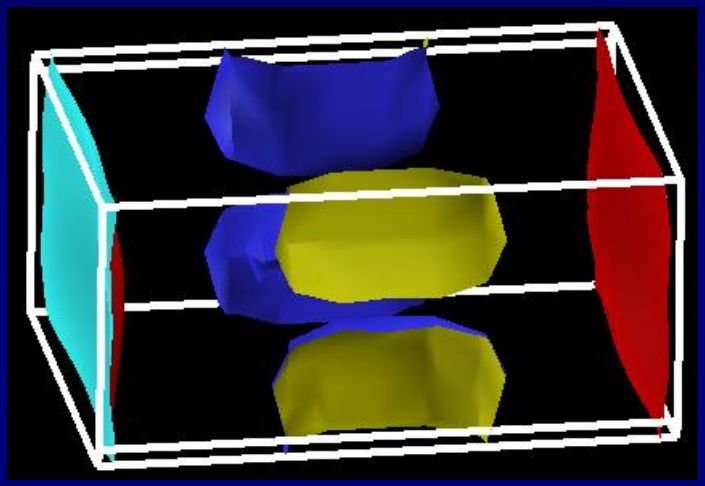
picene

青木 秀夫: グラフェンの物理

K_2K_1picene

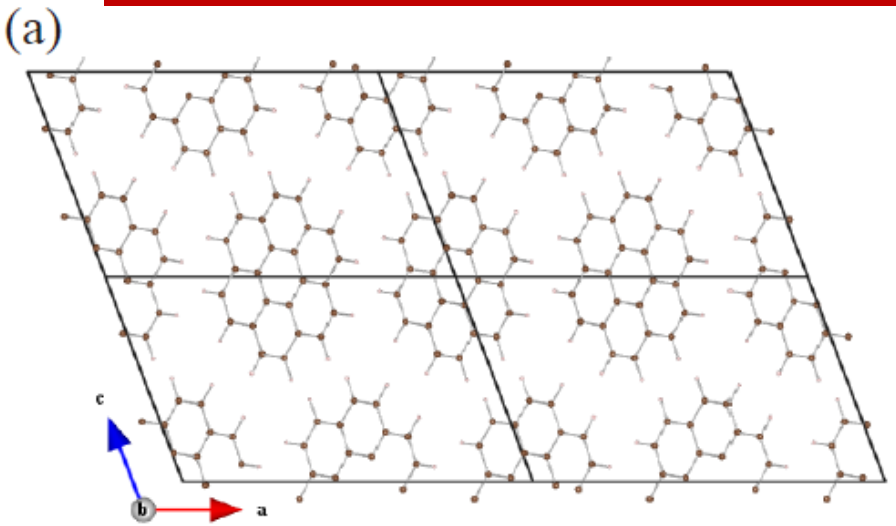
Kosugi et al（2009）

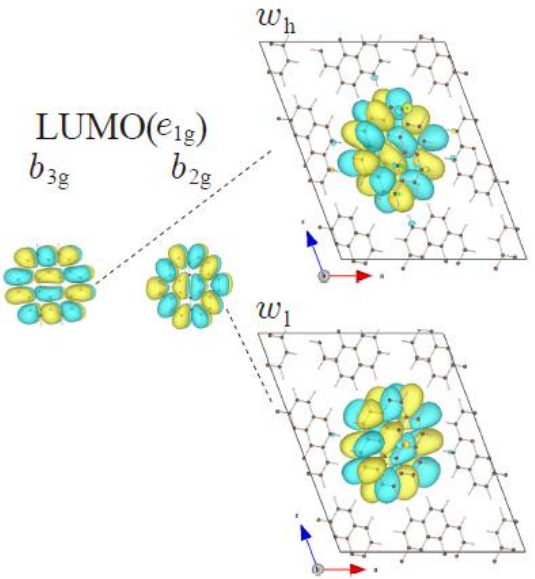


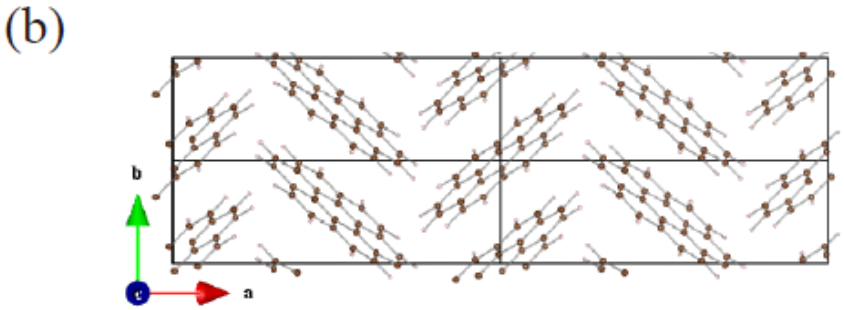


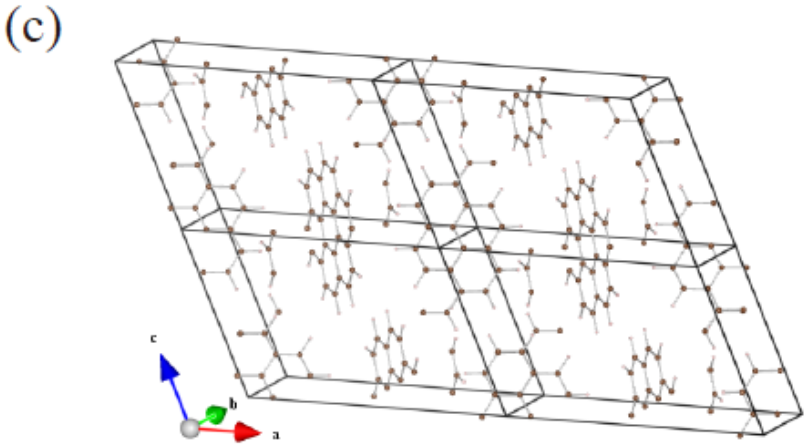
Electronic str of solid coronene

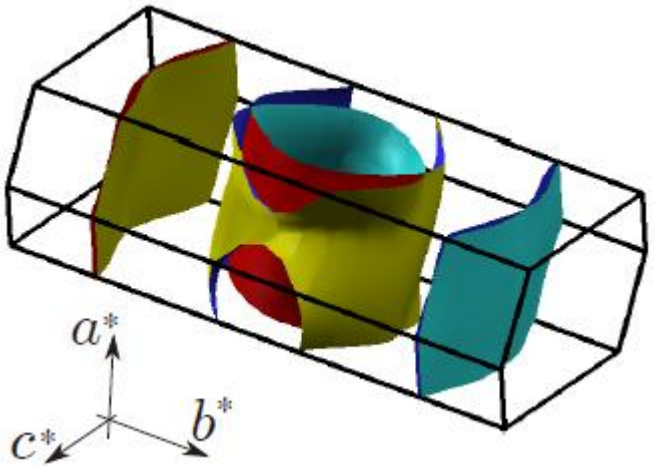
(Kosugi et al, arXiv:1105.0248)









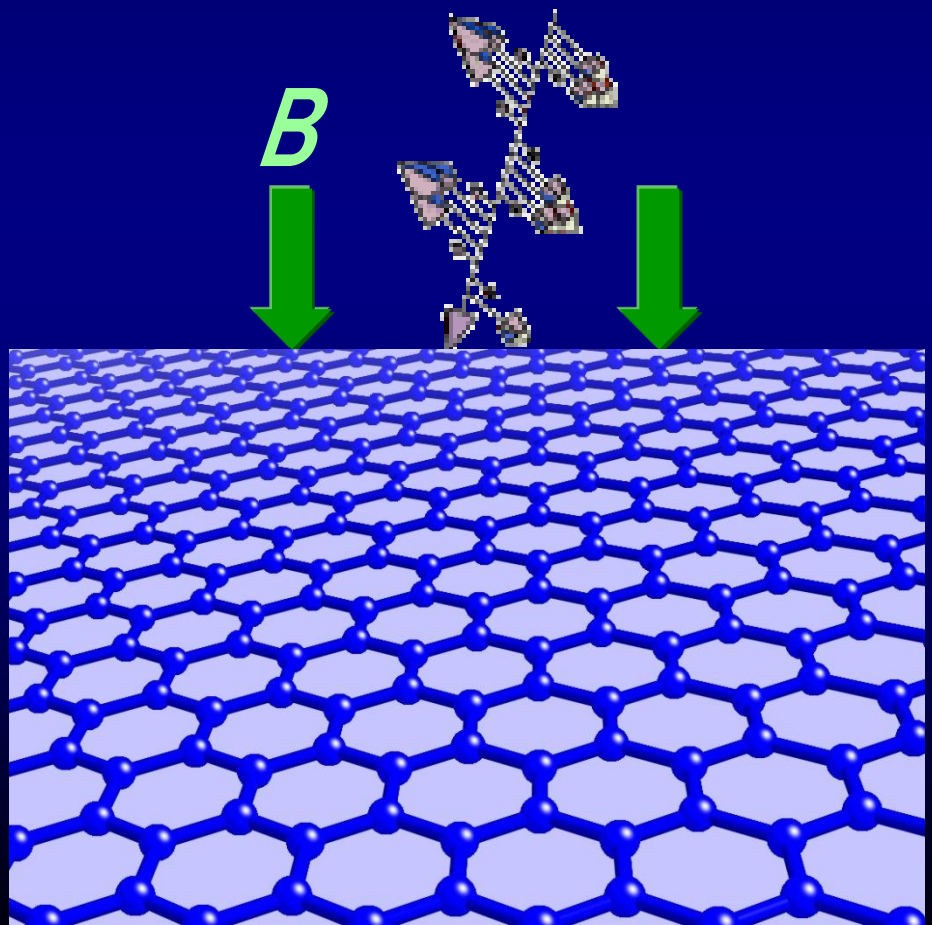


青木 秀夫: グラフェンの物理

AC効果

dc QHE → ac (optical) QHE ?

(Morimoto, Hatsugai & Aoki, PRL 2009;
Ikebe et al, PRL 2010)



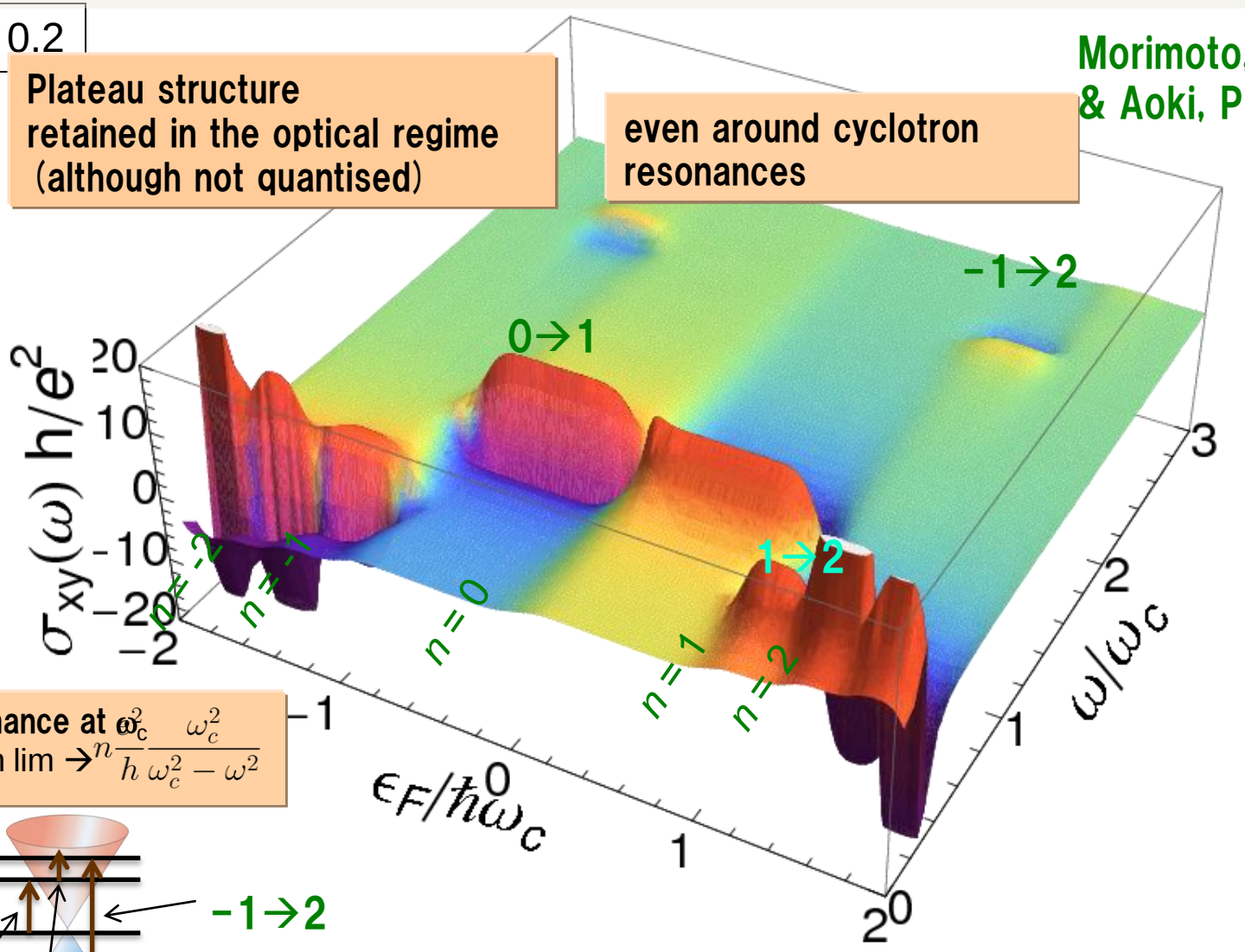
Optical Hall conductivity for Dirac QHE

$\Gamma = 0.2$

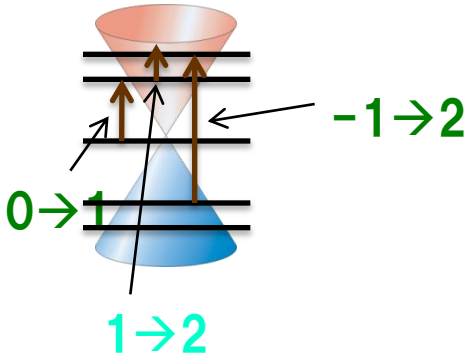
Plateau structure
retained in the optical regime
(although not quantised)

even around cyclotron
resonances

Morimoto, Hatsugai
& Aoki, PRL 2009



Resonance at ω_c^2
(clean lim → $n\frac{\omega_c^2}{\hbar}\frac{\omega_c^2}{\omega_c^2-\omega^2}$



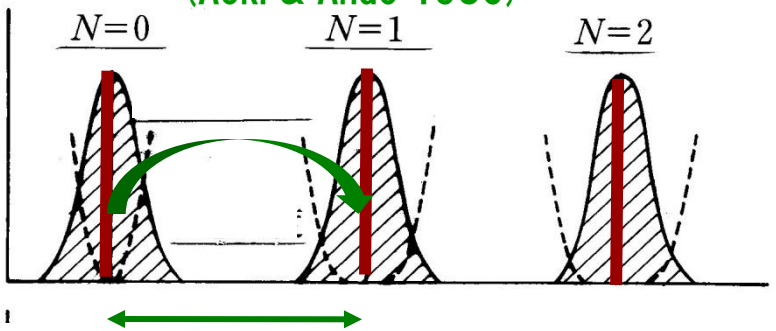
$$\sigma_{xy} = -i\frac{1}{L^2}\sum_{\alpha,\beta} f(\varepsilon_\alpha)\frac{\langle\alpha|J_x|\beta\rangle\langle\beta|J_y|\alpha\rangle - \langle\alpha|J_y|\beta\rangle\langle\beta|J_x|\alpha\rangle}{(\varepsilon_\alpha-\varepsilon_\beta)^2}$$

青木 秀夫: グラフェンの物理

Why the plateaus robust even in ac ?

Localisation →
transitions mainly between
extended states

(Aoki & Ando 1980)

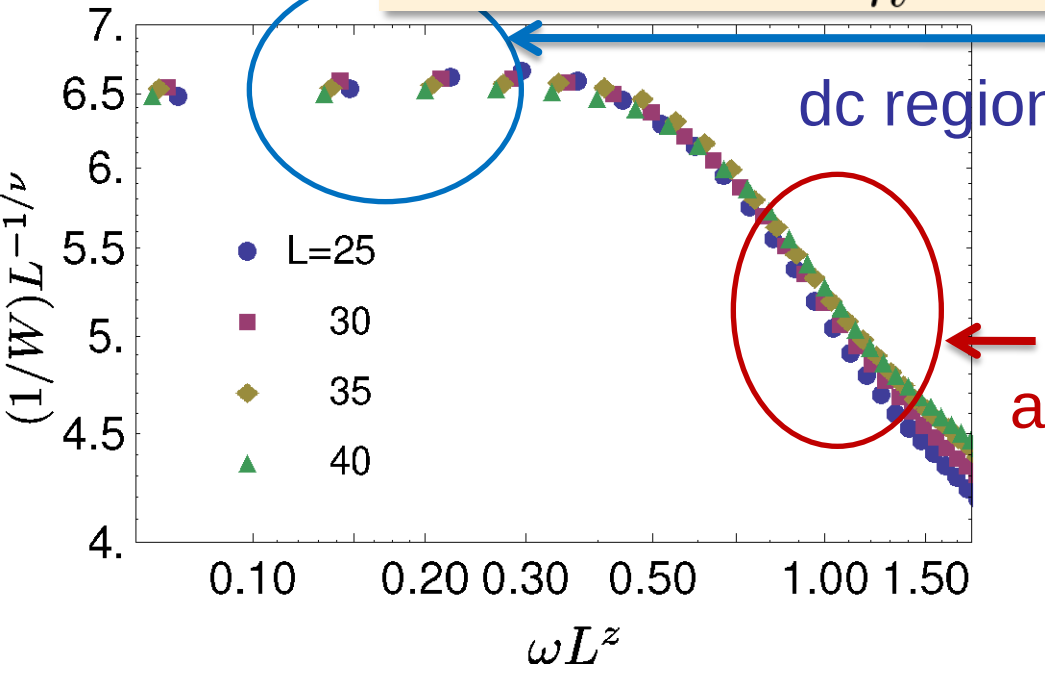


$\sigma_{xy}(\omega)$

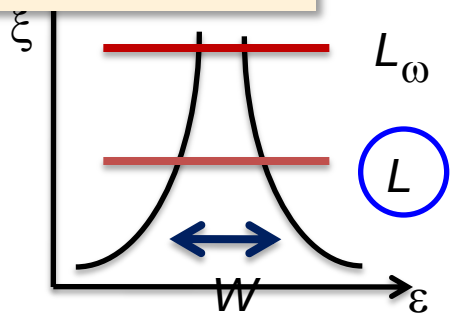
$$= \frac{e^2\hbar}{iL^2} \sum_{\epsilon_a<E_F} \sum_{\epsilon_b\geq E_F} j_x^{ab} j_y^{ba} \frac{1}{\hbar\omega} \left(\frac{1}{\epsilon_b - \epsilon_a - \hbar\omega} - \frac{1}{\epsilon_b - \epsilon_a} \right) - j_y^{ab} j_x^{ba} \frac{1}{\hbar\omega} \left(\frac{1}{\epsilon_b - \epsilon_a} - \frac{1}{\epsilon_b - \epsilon_a + \hbar\omega} \right)$$

Dynamical scaling for $\sigma_{xy}(\omega)$

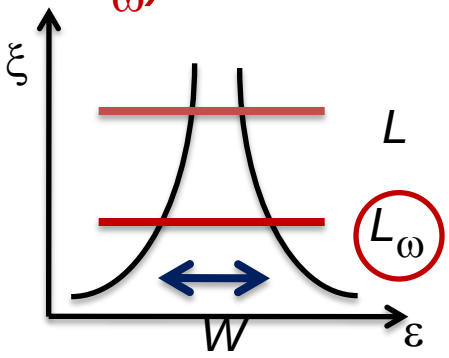
$$\sigma_{xy}(\varepsilon_F, \omega, L) = \frac{e^2}{h} F((\varepsilon_F - \varepsilon_c) L^{1/\nu}, \omega L^z)$$



dc region ($L < L_\omega$)



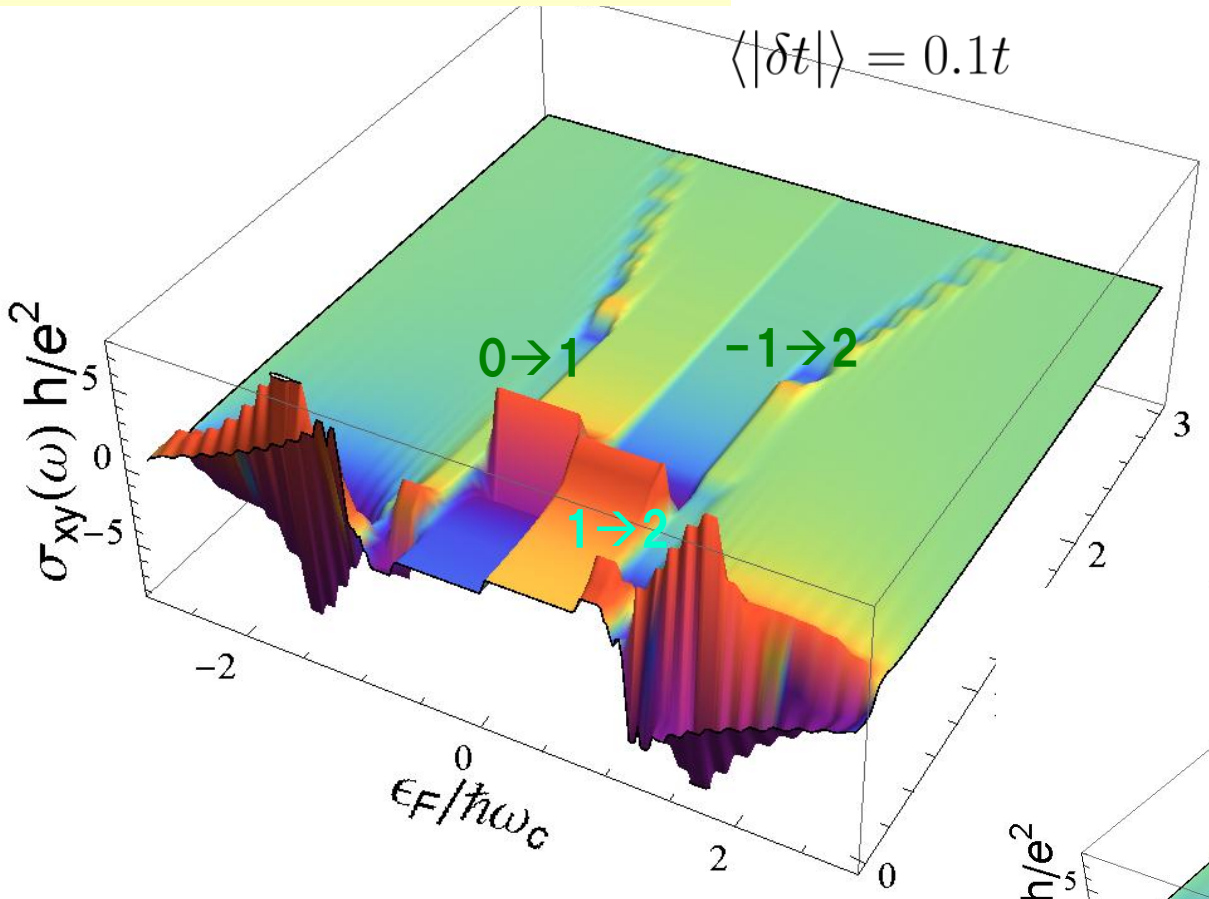
ac region ($L > L_\omega$)



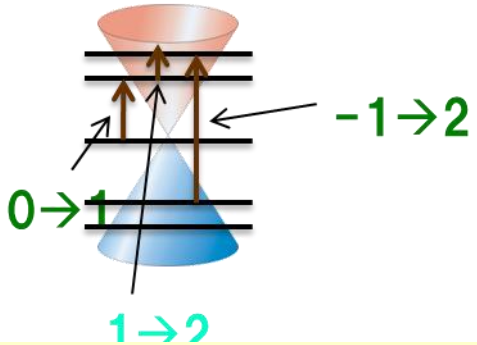
Dirac → honeycomb lattice

Morimoto, Hatsugai & Aoki, PRL 2009;
Kawarabayashi et al, PRB 2010

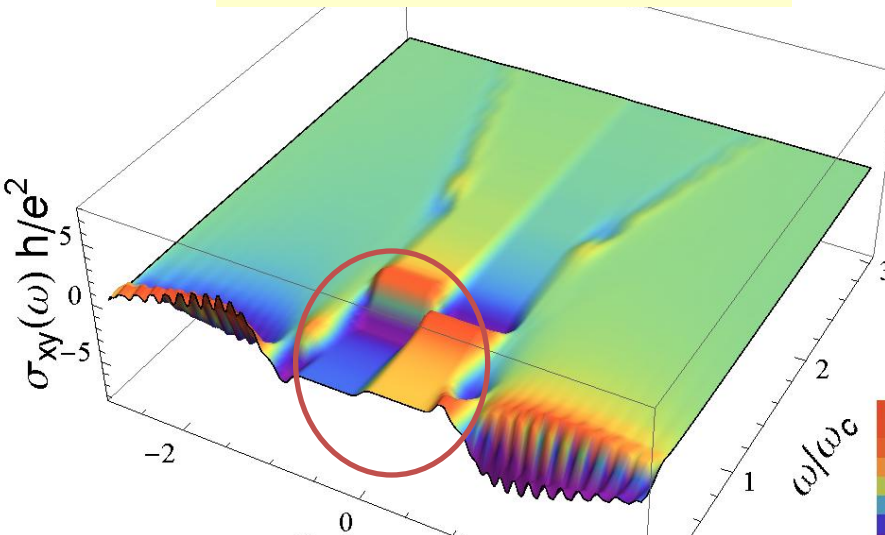
Chiral symmetric bond disorder



Resonance at $\omega_c^2 \quad \omega_c^2$
(clean lim →$^n \frac{}{h\,\omega_c^2 - \omega^2}$



Non-chiral symmetric potential disorder



t

$\sigma_{xy}(\omega)$ observable as Faraday rotation

Faraday rotation ∝ optical Hall conductivity

$$\Theta_H(\omega) \sim \frac{1}{(n_0+n_s)c\epsilon_0}\sigma_{xy}(\omega)$$

plateau

$$\cong \frac{1}{(n_0+n_s)c\epsilon_0}\frac{e^2}{h}$$

$$\sim \frac{2}{n_0+n_s}\alpha \sim 10\ \text{mrad}$$

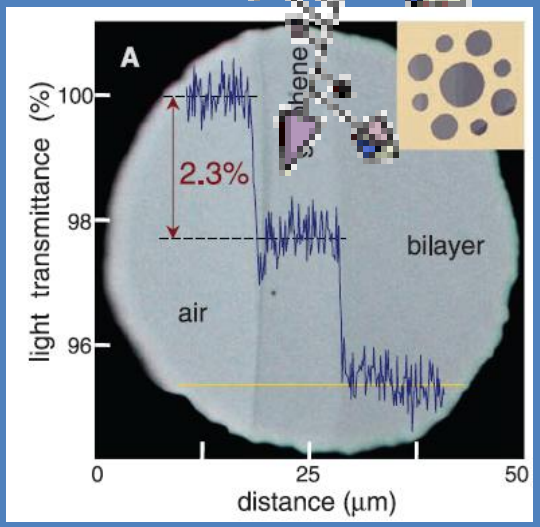
n_0: air, n_s: substrate

Resolution ~ 1 mrad
(Ikebe, Shimano, APL
→ exp feasible !

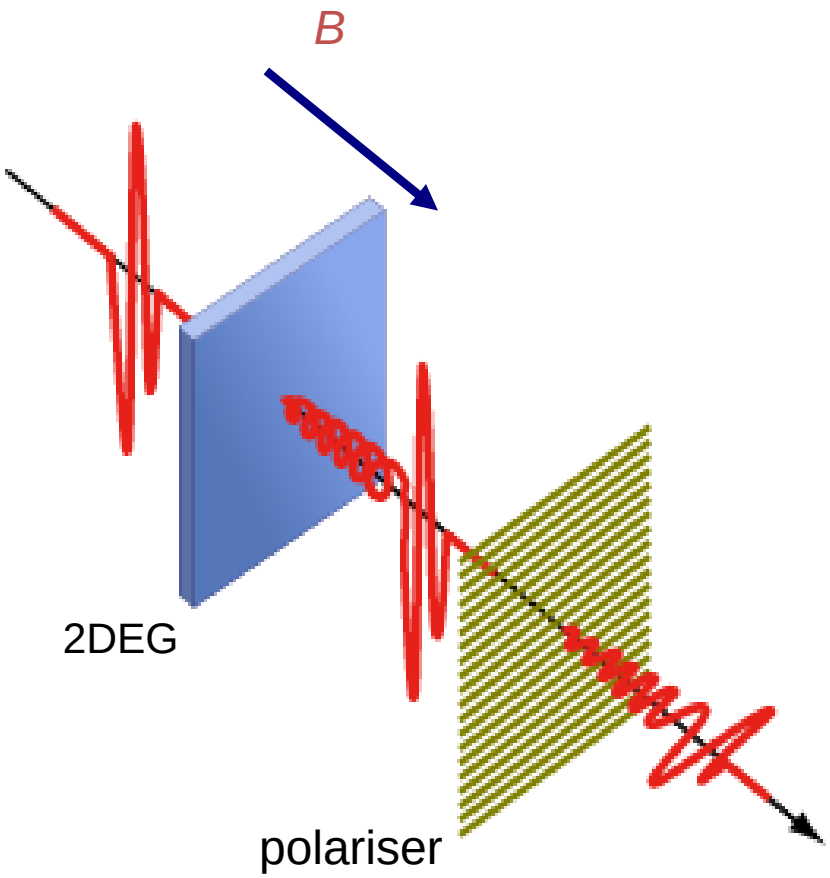
Faraday rotation ~ fine structure constant:
"α seen as a rotation"

Fine Structure Constant Defines Visual Transparency of Graphene

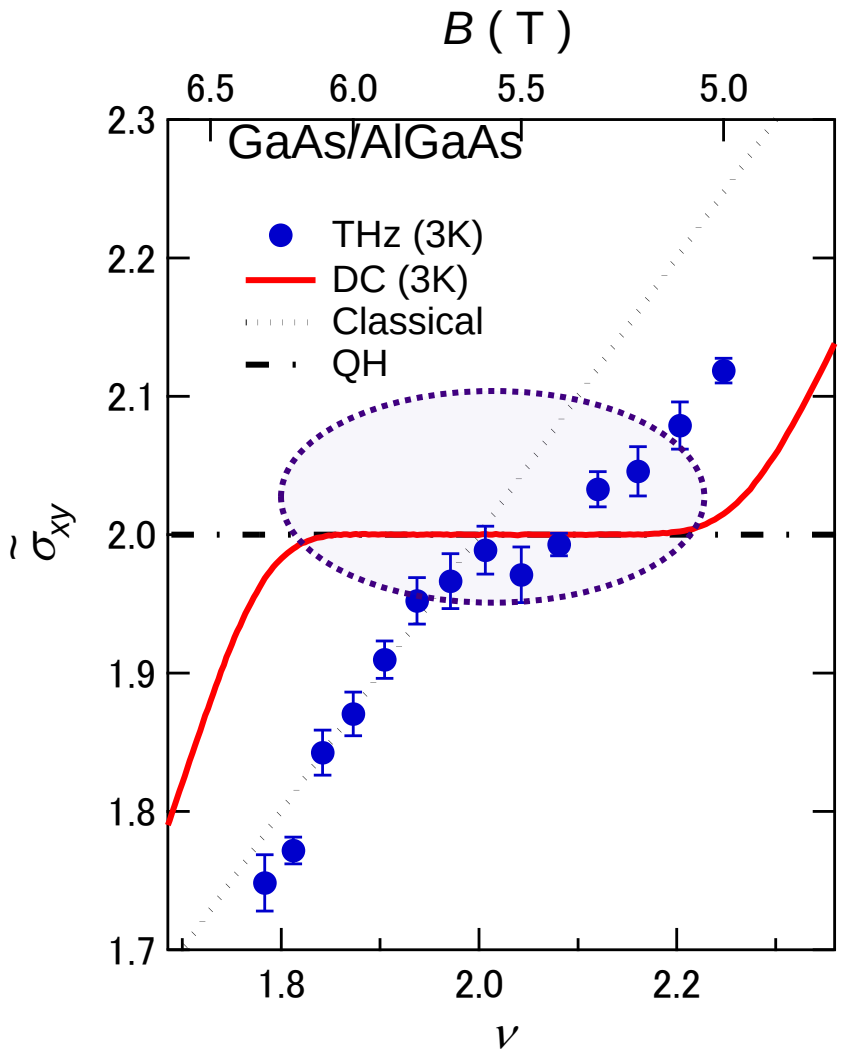
(Nair et al, Science 320)



THz detection of plateau in 2DEG



Ikebe et al, PRL 2010

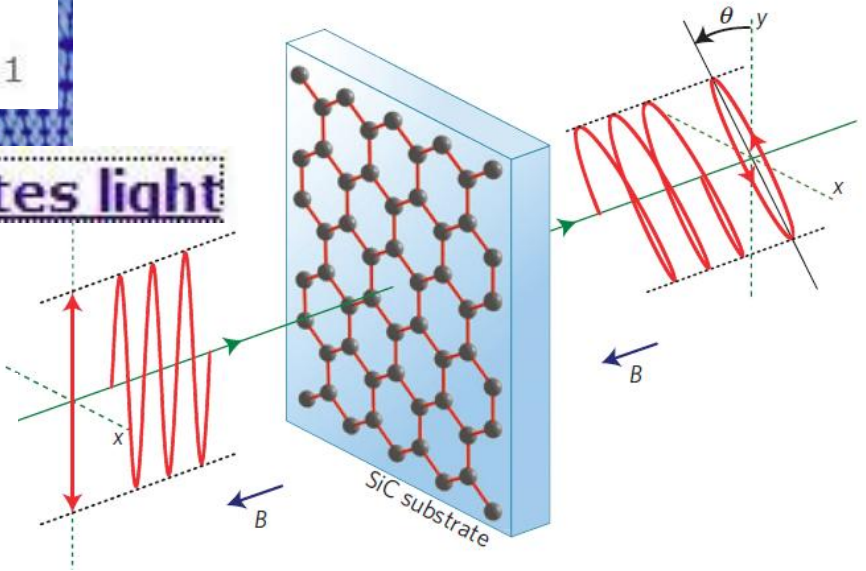


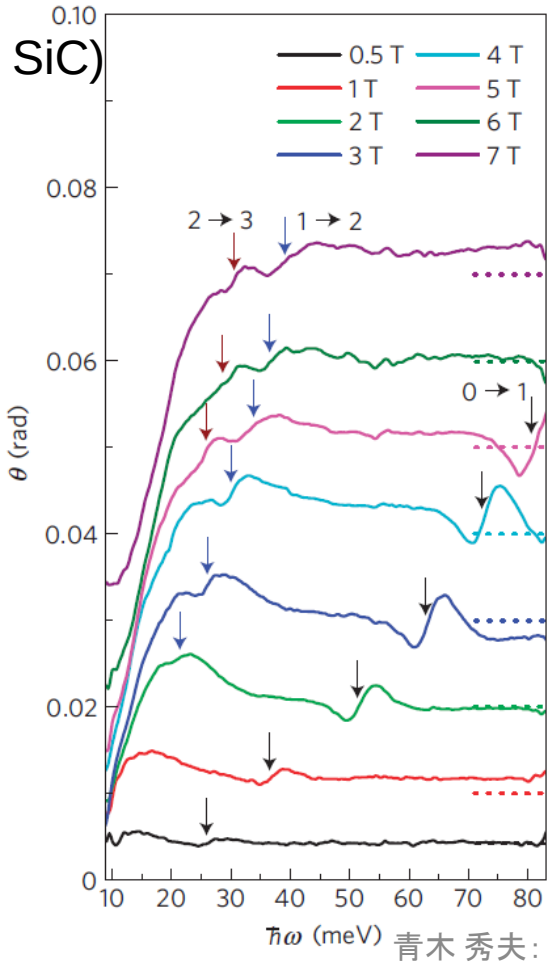
nature physics
Current issue
January 2011 - Vol 7 No 1

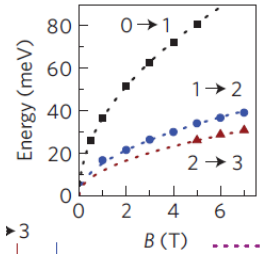
Faraday effect: Graphene rotates light

(Crassee et al, nature phys 2010)

"Multi-layer" (epitaxial from SiC) graphene





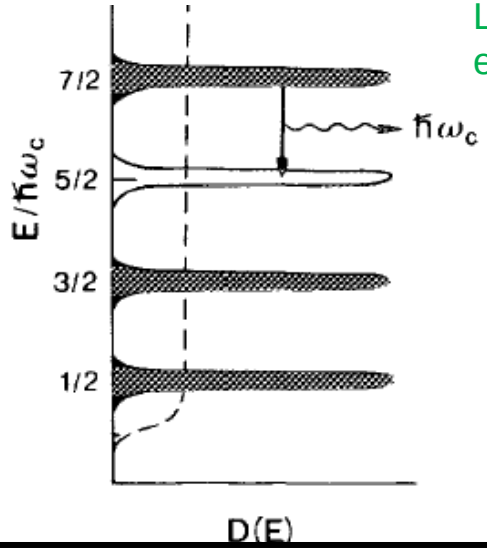


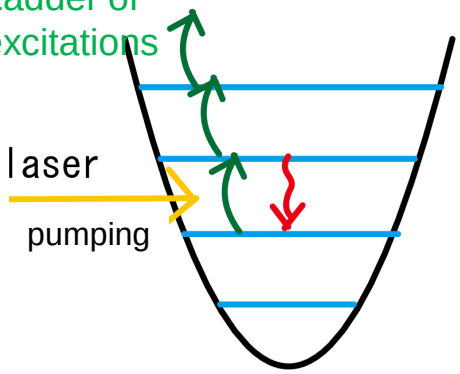
Cyclotron emission in graphene QHE

Ordinary QHE systems (2DEG)

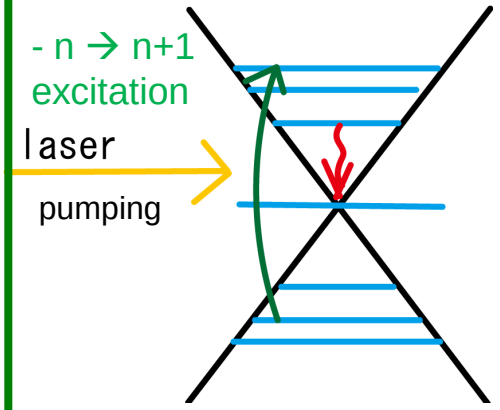
Landau-level laser

(Aoki, APL 1986)





Graphene QHE system



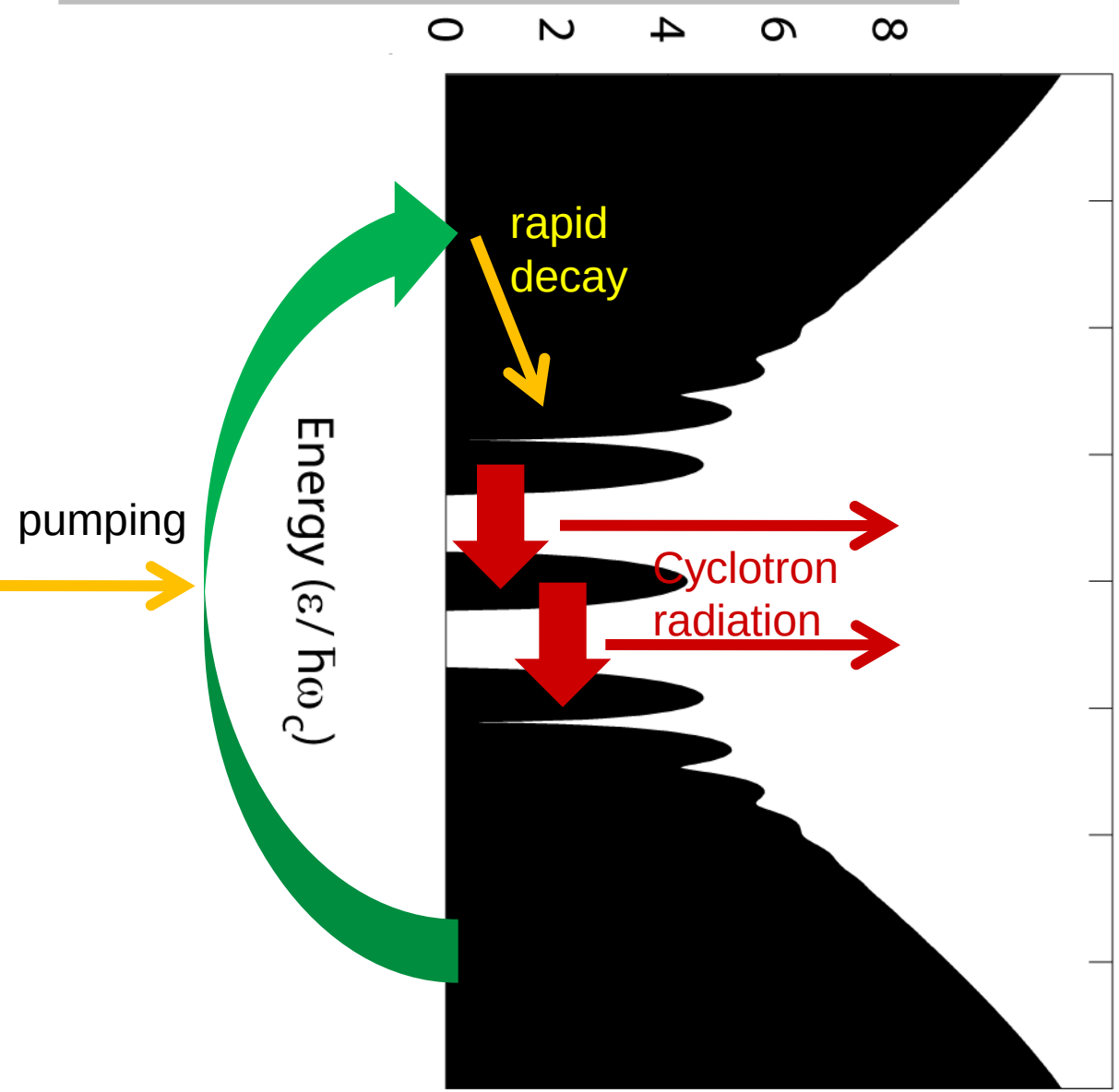
(Morimoto, Hatsugai & Aoki, 2007)

→ **Photon emission rate**

2DEG: ～ B^2 << graphene: ～ $\sqrt{B}$

青木 秀夫: グラフェンの物理

n=0 Landau level



Uneven Landau levels

↓

n =0 Landau level can stand alone

↓

Cyclotron radiation from continuum to *n* =0 enhanced

➔ Photoemission rate
vs
relaxation due to phonons
--- should be examined

青木 秀夫: グラフェンの物理

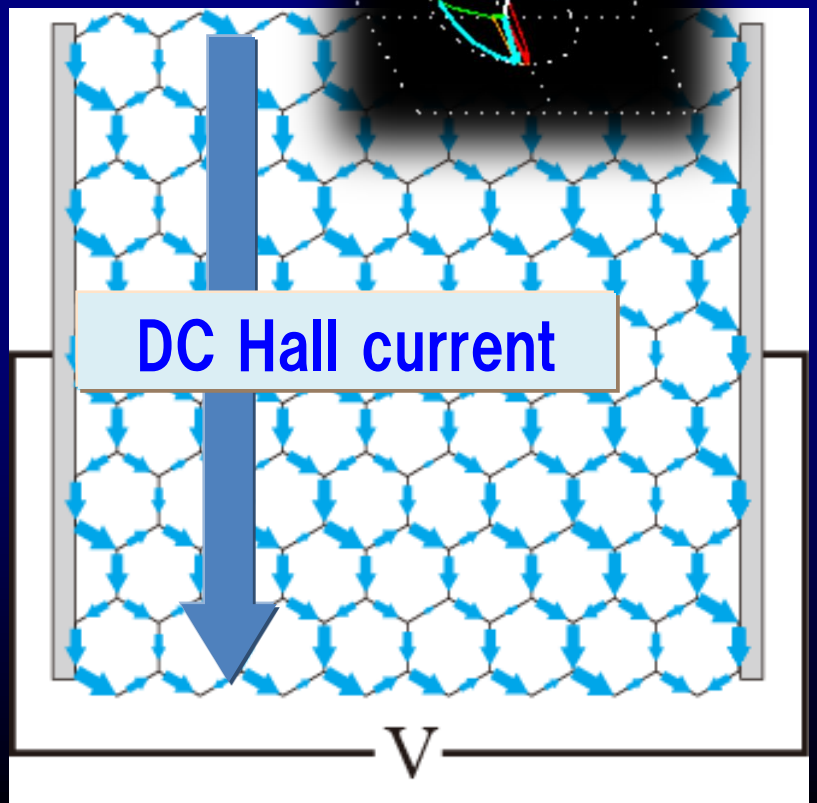
非平衡とトポロジカルな性質

Photovoltaic Hall effect

Circularly-polarised light in B = 0
→ breaks time-reversal

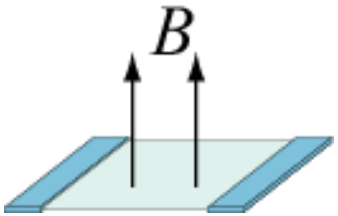
(Oka & Aoki, PRB 2009)

$B = 0$

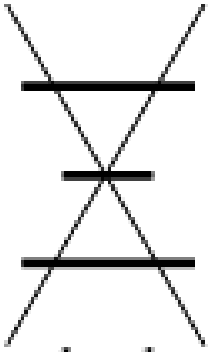


Hall effects in Dirac systems

1. QHE



graphene



Landau level

(Geim; Kim)

2. Spin Hall effect in topological insulators



+ spin-orbit

$$H = \begin{pmatrix} m_{\uparrow,\downarrow} & k_x - ik_y \\ k_x + ik_y & -m_{\uparrow,\downarrow} \end{pmatrix} \qquad \sigma_{xy} = \frac{1}{2}\frac{e^2}{2\pi}\frac{m_{\uparrow,\downarrow}}{|m_{\uparrow,\downarrow}|}$$

mass term ← spin-orbit (Kane & Mele, PRL 2005)

Spin-orbit too small in graphene; rather, HgTe systems

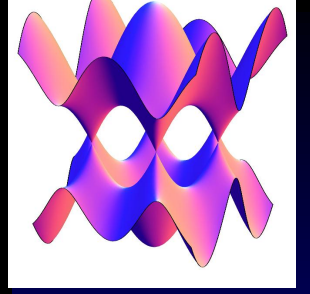
3. Photovoltaic Hall effect



Circularly-polarised light
→ breaks time-reversal

(Oka & Aoki, PRB 2009)

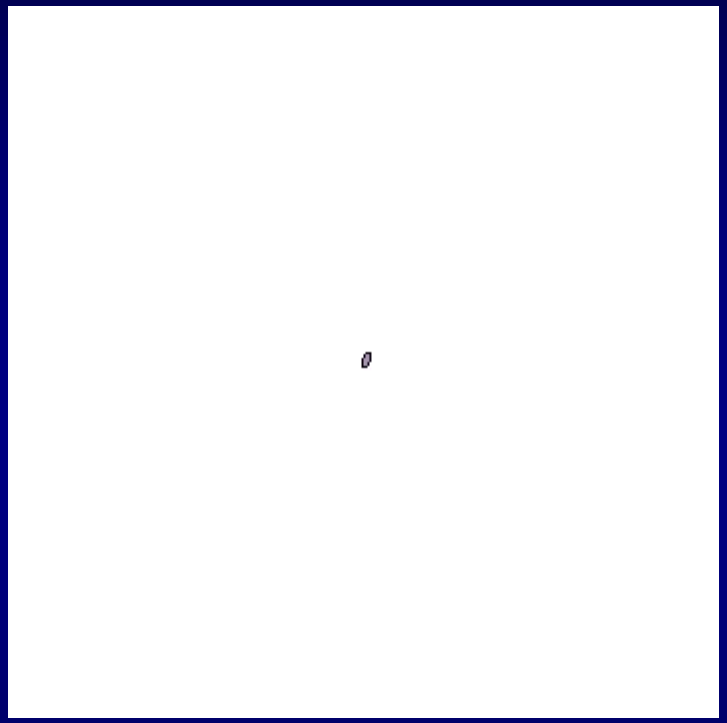
Wave propagation in graphene

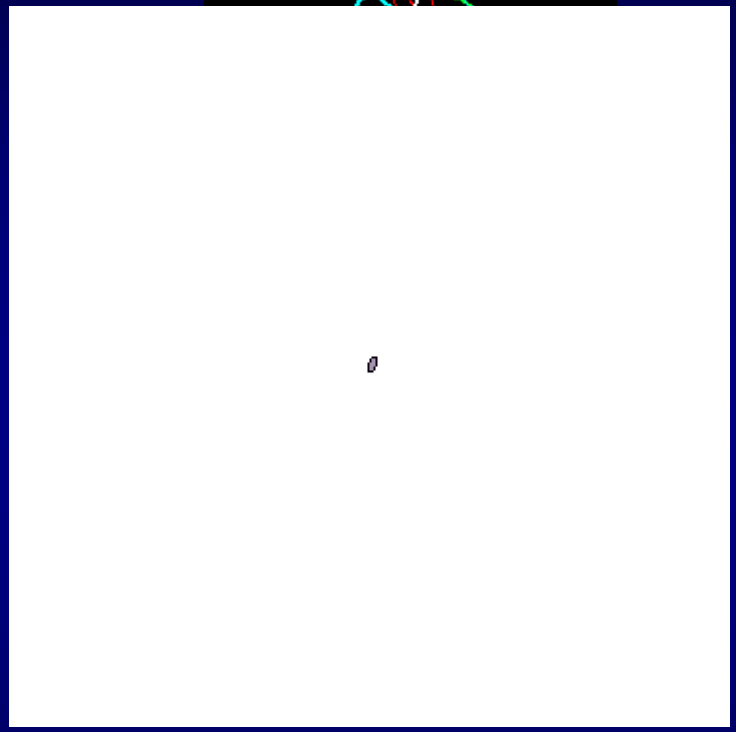


$B = 0$



$B = 0$





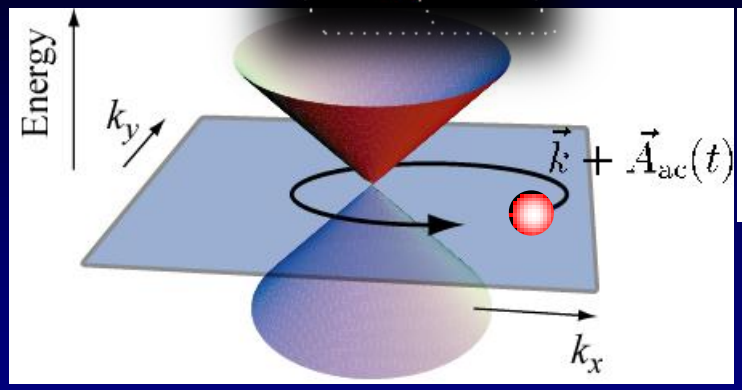
$R(t) \sim vt$

Animation by Oka

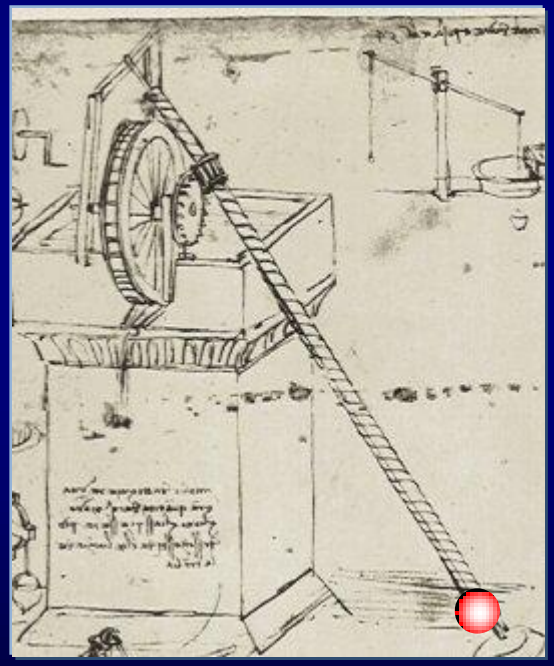
$R(t) \sim t^{1/2}$

Dynamical mass → gap

Why a DC response in an AC field? – Geometric phase

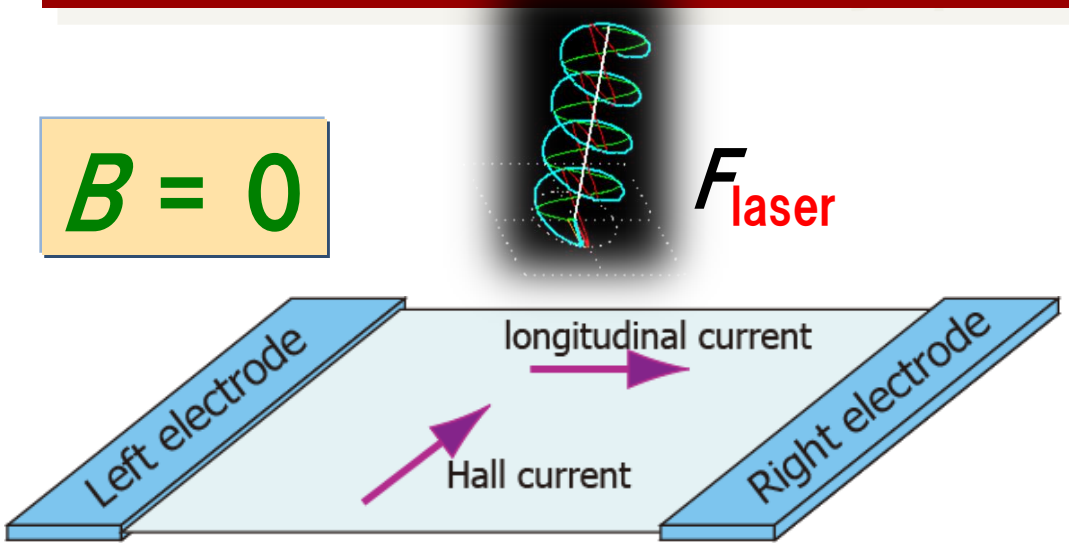


→ Aharonov-Anandan phase (PRL 1987)
→ Non-adiabatic charge pumping
→ Photovoltaic Hall effect



Photovoltaic DC Hall effect in graphene in a circularly polarised light

B = 0



(Oka & Aoki, 2009)

$$J_{\text{dc}}^{\text{Hall}} = \sigma_{xy} E_{\text{electrode}}^{x}$$

$$\sigma_{xy} \propto F_{\text{laser}}^{2}$$

- Th framework:

  Kubo-formula for DC transport + Keldysh formalism

$$\sigma_{ab} = i \int \frac{d\boldsymbol{k}}{(2\pi)^d} \sum_{\alpha,\beta\neq\alpha} \frac{[f_\beta(\boldsymbol{k}) - f_\alpha(\boldsymbol{k})]}{E_\beta(\boldsymbol{k}) - E_\alpha(\boldsymbol{k})} \frac{\langle\Phi_\alpha(\boldsymbol{k})|J_b|\Phi_\beta(\boldsymbol{k})\rangle\langle\Phi_\beta(\boldsymbol{k})|J_a|\Phi_\alpha(\boldsymbol{k})\rangle}{E_\beta(\boldsymbol{k}) - E_\alpha(\boldsymbol{k}) + i\eta}$$

used in Aoki & Ando (1981) for the static QHE

Here, we need to take account of the effect of strong AC background
→ we can achieve this with the Floquet formalism

Floquet theorem
(temporal analogue of the Bloch theorem)

Time-periodic Hamiltonian ⟹ periodic solution = Floquet state

$$H(t+T) = H(t)$$

$$|\Psi_\alpha(t)\rangle \equiv e^{-i\varepsilon_\alpha t}|\Phi_\alpha(t)\rangle, \qquad |\Phi_\alpha(t+T)\rangle = |\Phi_\alpha(t)\rangle$$

$$\varepsilon = \frac{1}{T}\int_0^T E(t)dt + \gamma^{\mathrm{AA}}/T$$

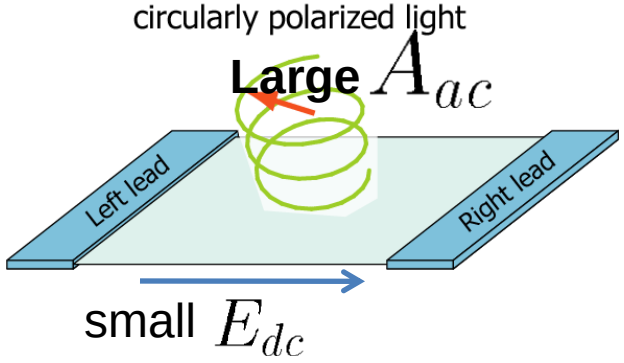
dynamical phase

Aharonov-Anandan phase

$$\gamma_\alpha^{\mathrm{AA}} \equiv T\langle\langle\Phi_\alpha|i\partial_t|\Phi_\alpha\rangle\rangle = \pm\pi\{[4(A/\Omega)^2+1]^{-1/2}-1\}$$

With the Floquet basis we can extend Kubo formula
(a) Photon-dressed Floquet states
(b) Aharonov-Anandan phase emerges

Kubo formula in strong AC fields (general)



$$J^i_{\rm dc} = \sigma_{ij}(\boldsymbol{A}_{\rm ac}) E^j_{\rm dc}$$

$$\sigma_{ab}(\boldsymbol{A}_{ac}) = i \int \frac{d\boldsymbol{k}}{(2\pi)^d} \sum_{\alpha,\beta\neq\alpha} \frac{[f_\beta(\boldsymbol{k}) - f_\alpha(\boldsymbol{k})]}{\varepsilon_\beta(\boldsymbol{k}) - \varepsilon_\alpha(\boldsymbol{k})} \frac{\langle\langle \Phi_\alpha(\boldsymbol{k})|J_b|\Phi_\beta(\boldsymbol{k})\rangle\rangle \langle\langle \Phi_\beta(\boldsymbol{k})|J_a|\Phi_\alpha(\boldsymbol{k})\rangle\rangle}{\varepsilon_\beta(\boldsymbol{k}) - \varepsilon_\alpha(\boldsymbol{k}) + i\eta}$$

ε_α : Floquet's quasi-energy

f_α : occupation fraction

$\langle\langle \Phi_\alpha|\Phi_\beta\rangle\rangle = \frac{1}{T}\int_0^T \langle \Phi_\alpha(t)|\Phi_\beta(t)\rangle$: time-averaged inner prod

Oka and Aoki, arXiv:0807.4767;
similar expression: Torres, Kunold, PRB (2005)

TKNN (Thouless, Kohmoto, Nightingale, Nijs) exteded to systems in AC

$$\sigma_{xy}(\boldsymbol{A}_{\rm ac}) = e^2 \int \frac{d\boldsymbol{k}}{(2\pi)^d} \sum_\alpha f_\alpha(\boldsymbol{k}) \left[\nabla_{\boldsymbol{k}} \times \boldsymbol{\mathcal{A}}_\alpha(\boldsymbol{k})\right]_z$$

$$\boldsymbol{\mathcal{A}}_\alpha(\boldsymbol{k}) \equiv -i\langle\langle \Phi_\alpha(\boldsymbol{k})|\nabla_{\boldsymbol{k}}|\Phi_\alpha(\boldsymbol{k})\rangle\rangle$$

Floquet states

Oka and Aoki, 2009

particles in arbitrary ac fields --- Floquet's theorem

- 光（振動電場）などの時間的に周期的な外場（強度は任意）に対し成り立つ.

- 空間的に周期的な系で成り立ったBlochの定理
  → 時間に周期的な系でのanalogue.

$$i\frac{d}{dt}\Psi(t) = H(t)\Psi(t) \qquad H(t+\tau) = H(t)$$

- Floquet state: $\Psi_\alpha(t) = e^{-i\varepsilon_\alpha t}u_\alpha(t), \qquad u_\alpha(t+\tau) = u_\alpha(t)$

- Fourier変換により、HamiltonianはFloquet 行列形式：

$$\sum_n H_{mn}\, u_\alpha^n = (\varepsilon_\alpha + m\Omega)\, u_\alpha^m$$

$$H_{mn} \equiv \frac{1}{\tau}\int_{-\pi/2}^{\pi/2} dt e^{i(m-n)\Omega t} H(t)$$

- 結局、時間依存の問題 → 時間に依存しない問題に置き換わった
  （代償としてFloquet mode n という自由が加わった）.

Floquet theorem (temporal analogue of the Bloch theorem)

Time-periodic Hamiltonian

$$H(t+T) = H(t)$$ ⟹ periodic solution = Floquet state

$$|\Psi_\alpha(t)\rangle \equiv e^{-i\varepsilon_\alpha t}|\Phi_\alpha(t)\rangle, \qquad |\Phi_\alpha(t+T)\rangle = |\Phi_\alpha(t)\rangle$$

$$\varepsilon = \frac{1}{T}\int_0^T E(t)dt + \gamma^{\mathrm{AA}}/T$$

dynamical phase — Aharonov-Anandan phase

TKNN (Thouless, Kohmoto, Nightingale, Nijs) exteded to systems in AC

Aharonov-Anandan curvature

$$\sigma_{xy}(\boldsymbol{A}_{\mathrm{ac}}) = e^2 \int \frac{d\boldsymbol{k}}{(2\pi)^d} \sum_\alpha f_\alpha(\boldsymbol{k}) \left[\nabla_{\boldsymbol{k}} \times \boldsymbol{\mathcal{A}}_\alpha(\boldsymbol{k})\right]_z$$

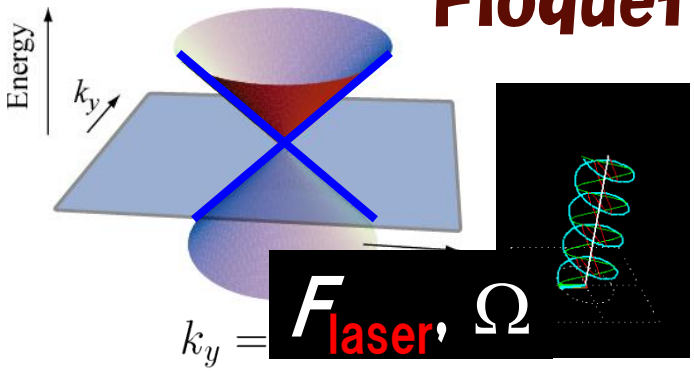
$$\boldsymbol{\mathcal{A}}_\alpha(\boldsymbol{k}) \equiv -i\langle\langle \Phi_\alpha(\boldsymbol{k})|\nabla_{\boldsymbol{k}}|\Phi_\alpha(\boldsymbol{k})\rangle\rangle$$

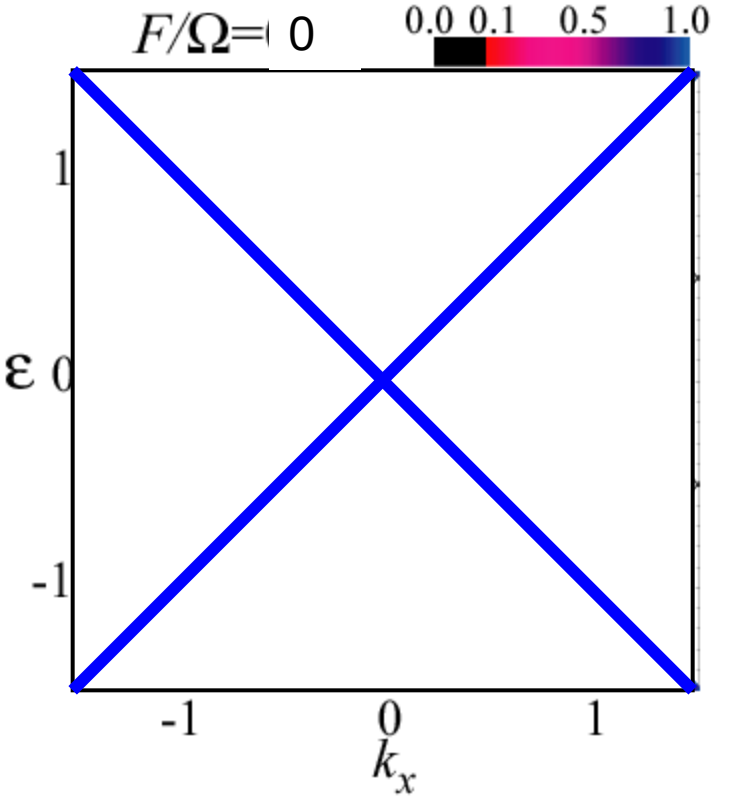
Floquet states (incl. AA phase)

(Oka & Aoki, 2009)

Floquet (photon-dressed) states in a Dirac band

(Oka & Aoki, 2009)





$$H^{mm-1} = \begin{pmatrix} 0 & 0 \\ A & 0 \end{pmatrix} \qquad A \equiv F/\Omega$$

$$\kappa = \frac{\sqrt{4A^2+\Omega^2}-\Omega}{2}$$

$$\kappa \propto F_{laser}^2/\Omega^3$$

→ dynamical gaps open
= AC Wannier-Stark ladder

Floquet quasi-energy ← dynamical gap opens

Photo-voltaic Hall effect in a Dirac band

(Oka & Aoki, 2009)

TKNN (Thouless, Kohmoto, Nightingale, Nijs) exteded to systems in AC

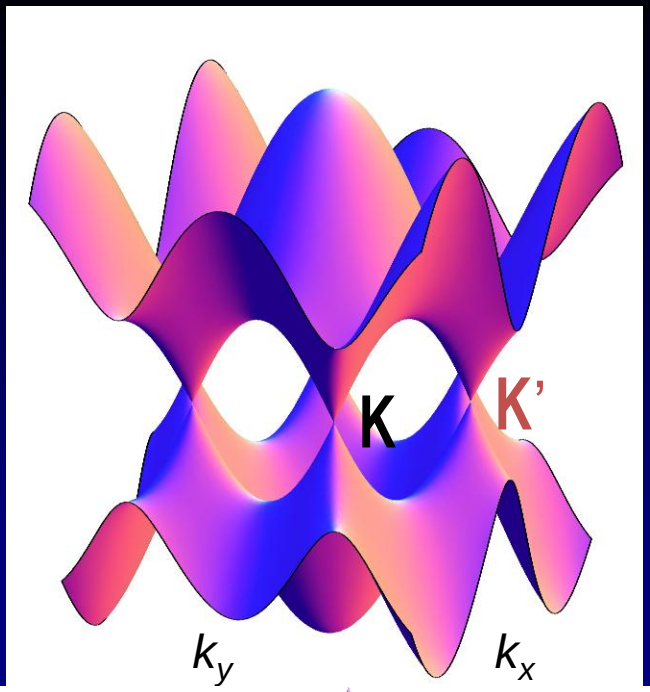
Aharonov-Anandan curvature

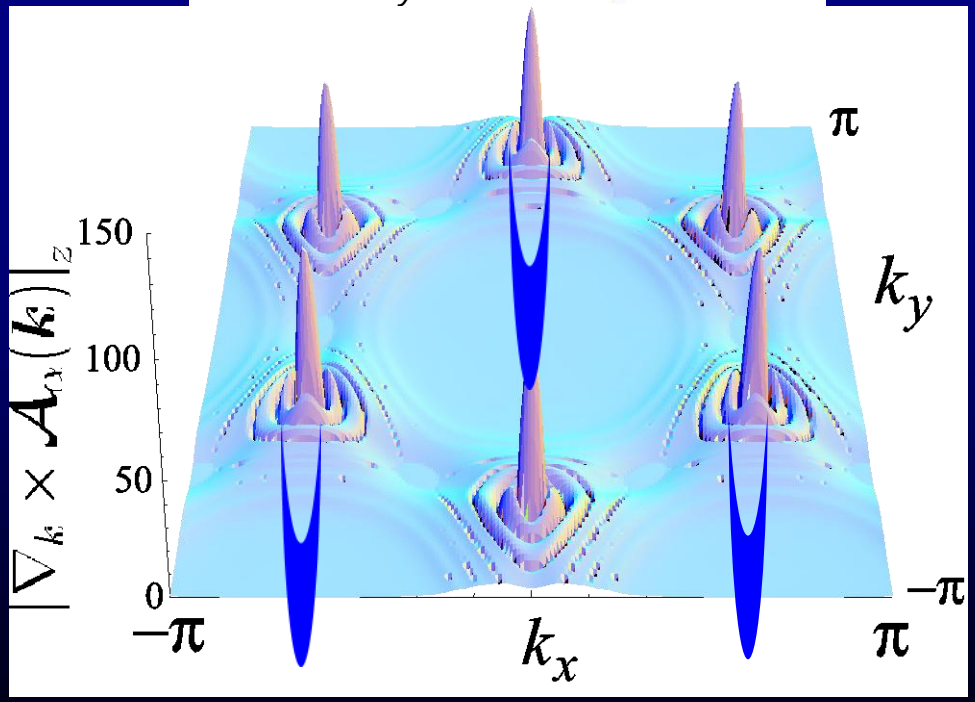
$$\sigma_{xy}(\boldsymbol{A}_{\rm ac}) = e^2 \int \frac{d\boldsymbol{k}}{(2\pi)^d} \sum_\alpha f_\alpha(\boldsymbol{k}) \left[\nabla_{\boldsymbol{k}} \times \boldsymbol{\mathcal{A}}_\alpha(\boldsymbol{k})\right]_z$$

$$\boldsymbol{\mathcal{A}}_\alpha(\boldsymbol{k}) \equiv -i\langle\langle \Phi_\alpha(\boldsymbol{k}) | \nabla_{\boldsymbol{k}} | \Phi_\alpha(\boldsymbol{k}) \rangle\rangle$$

Floquet states (incl. AA phase)

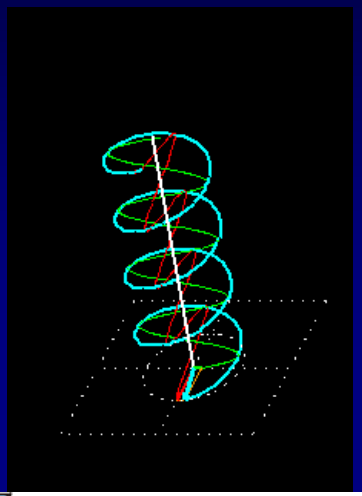
$$\left[\nabla_{\boldsymbol{k}} \times \boldsymbol{\mathcal{A}}_\alpha(\boldsymbol{k})\right]_z \sim \frac{1}{2}\kappa(|\boldsymbol{k}|^2 + \kappa^2)^{-3/2}$$

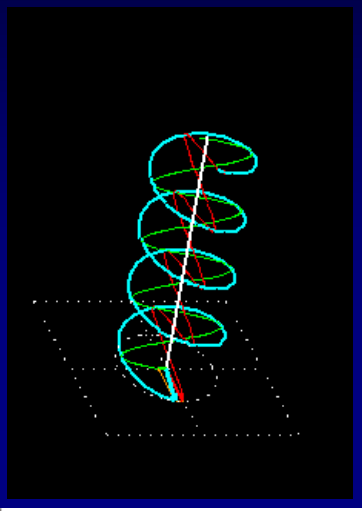


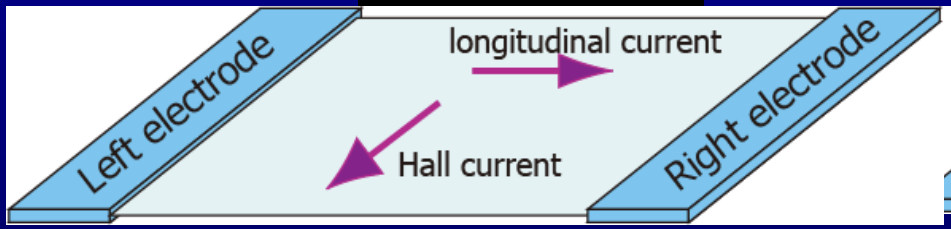


Nonequilibrium edge states also emerge (Lindner et al, nature phys 2011)

Photoinduced Hall = chirality control"







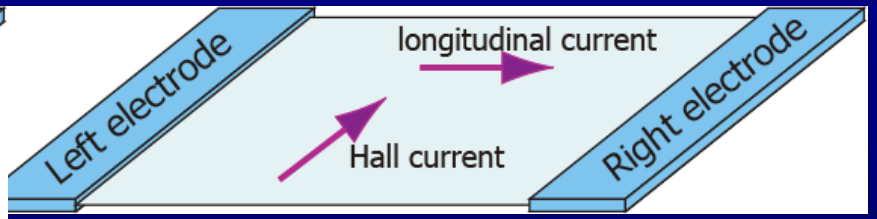
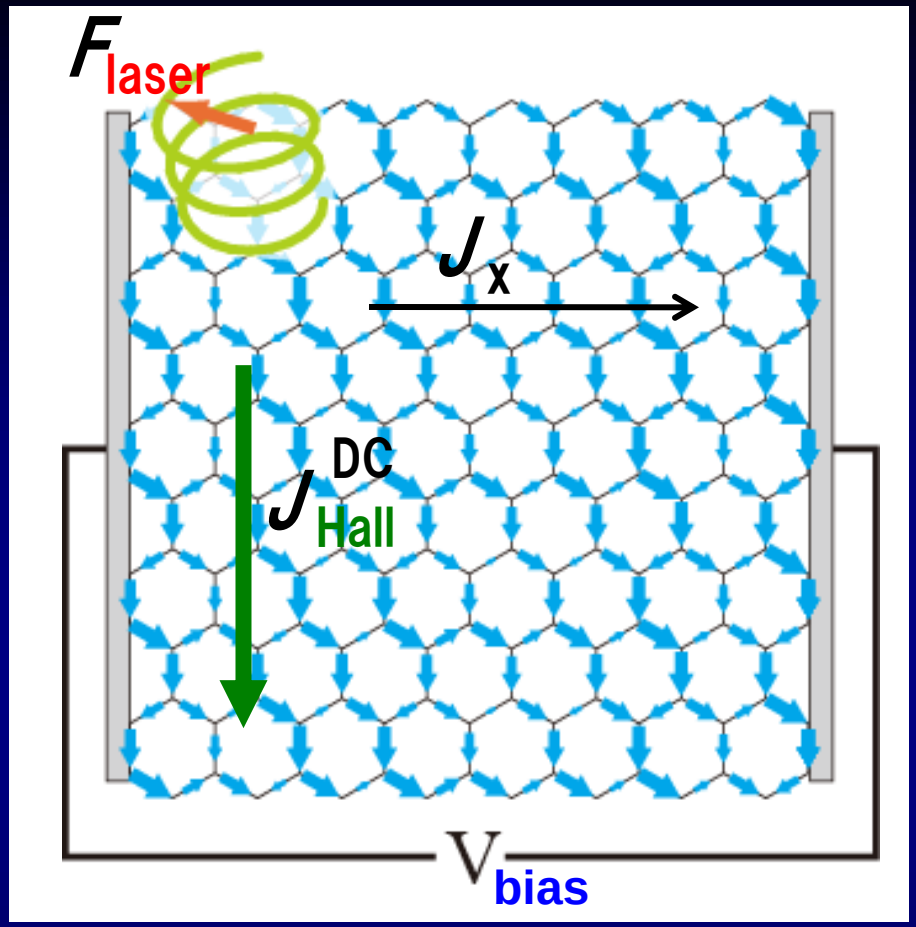
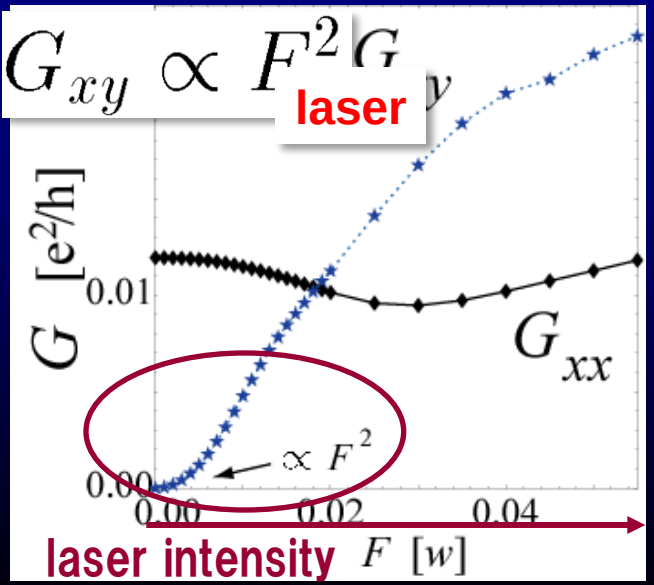


Photo-induced DC Hall conduction in an AC field

(Oka & Aoki, 2009)





Experimental feasibility

Required strength of the laser field:

E_{ac}~ 0.001 t (t = 2.7 eV: hopping energy)

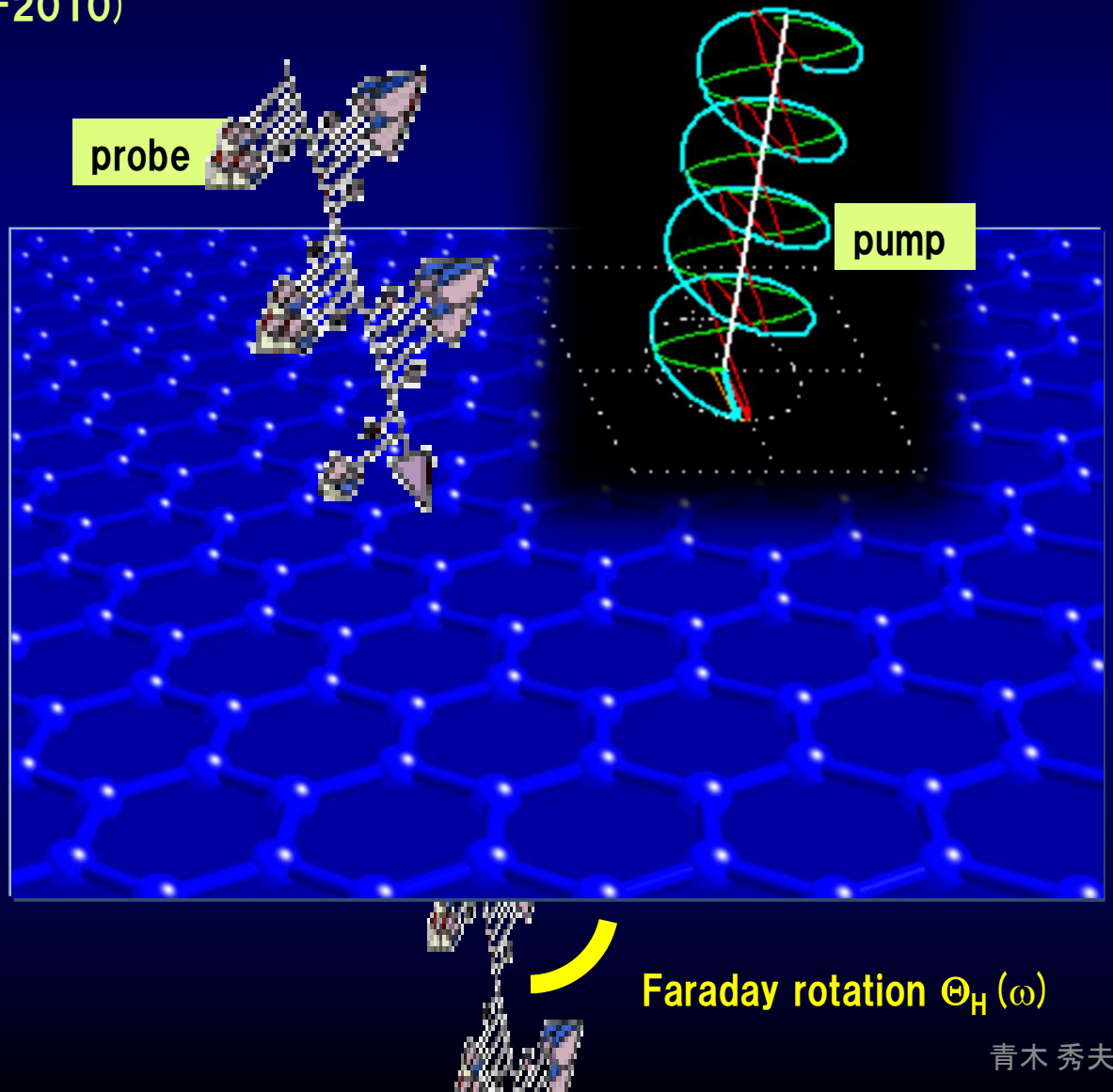
→ $E_{\mathrm{ac}} > 10^7 V/m$

for the laser photon energy $\Omega \sim 1\mathrm{eV}$

--- within the available intensity

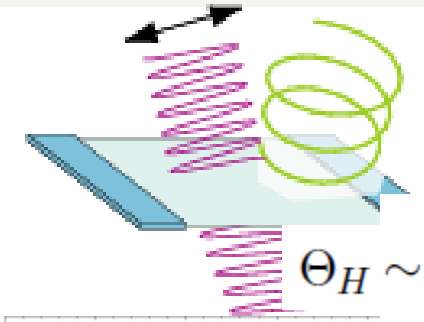
All-optical measurement: Faraday rotation

- Pump-probe spectroscopy
  (Oka & Aoki, HMF2010)

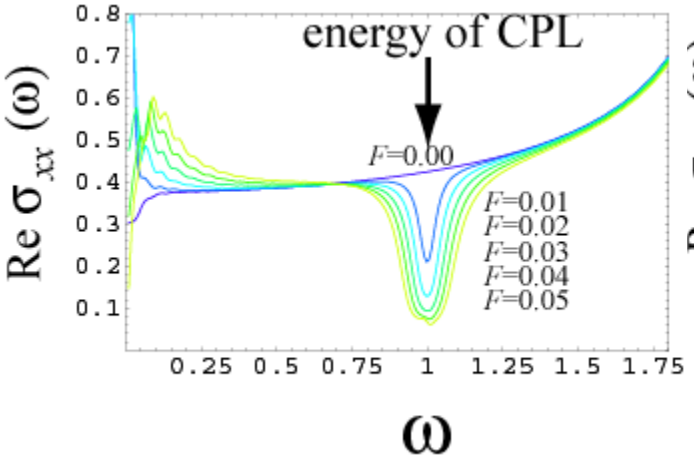


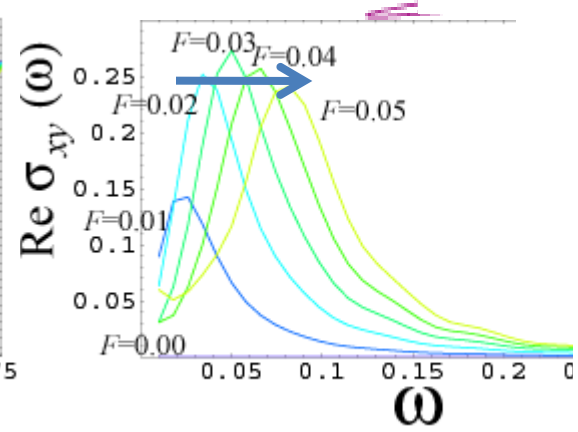
Faraday rotation as a nonlinear optical response

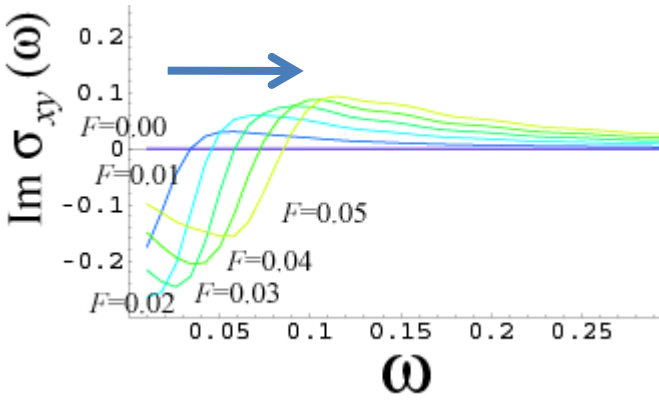
(Oka & Aoki, proc HMF 2010)



$$\Theta_H \sim (\sigma_{xy} \text{ in units of } \frac{e^2}{h}) \times 7 \text{ mrad}$$





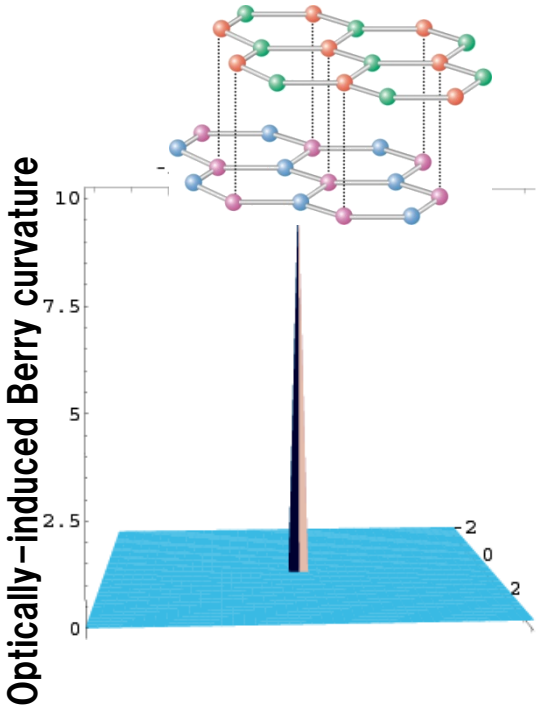


$$\Omega \sim 1 - 3eV$$
$$F \sim 10^7 V/m \quad \Rightarrow \quad \Theta_H \sim 1\text{mrad}$$

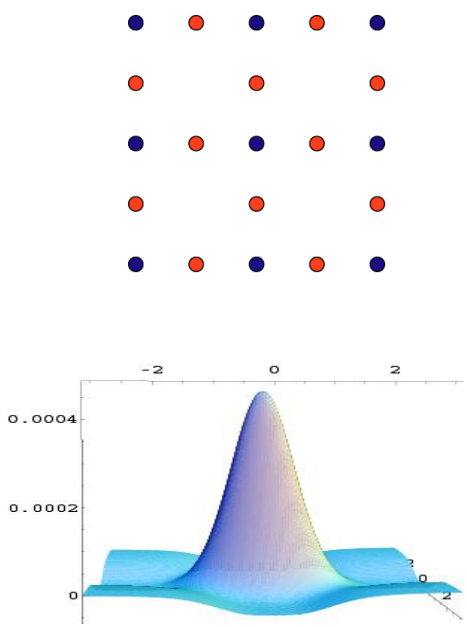
Other lattices?

(Oka & Aoki, proc HMF 2010)

Bilayer graphene



d-p model



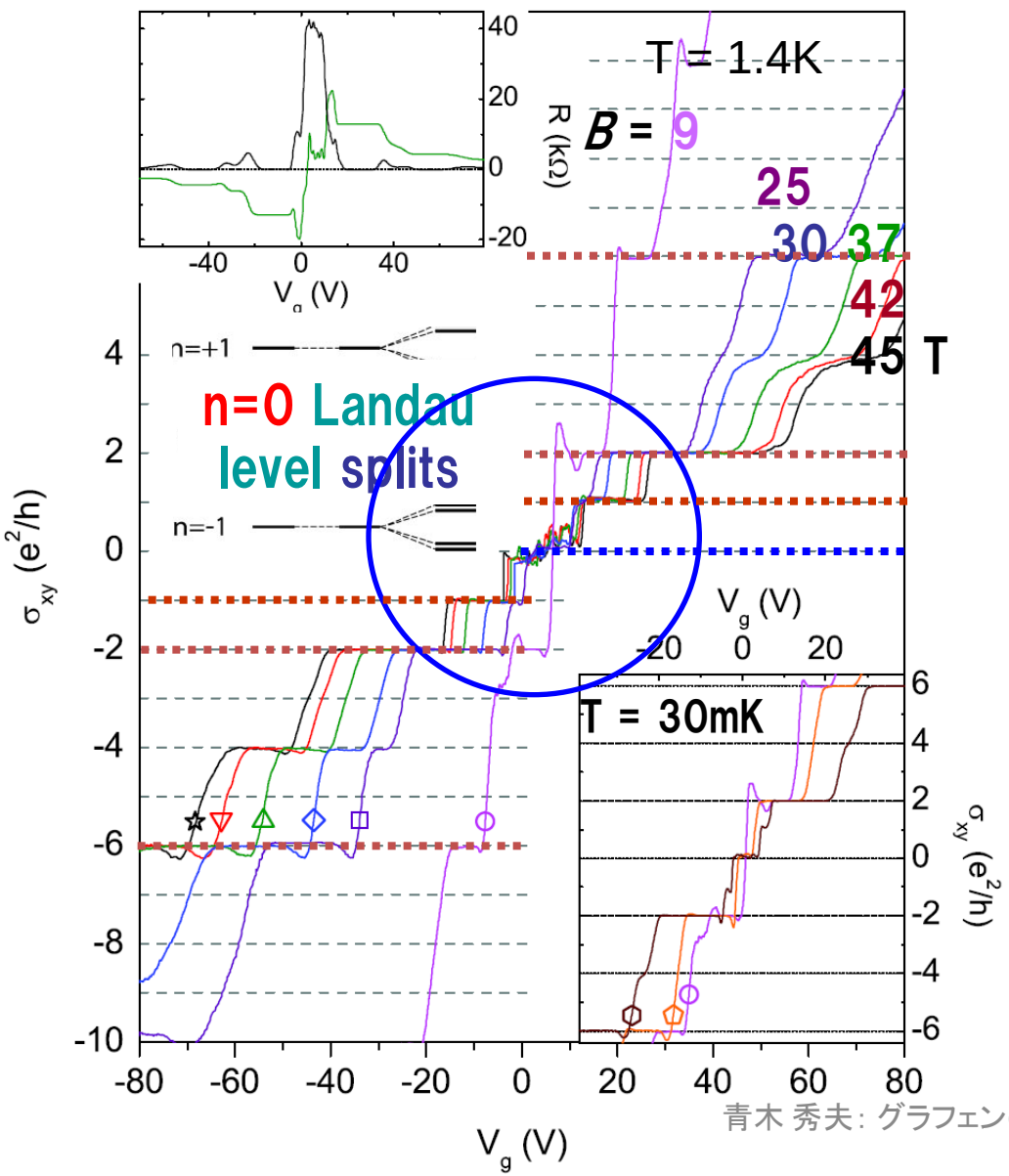
Graphite = single + bilayer graphene
(Koshino-Ando PRB2007)

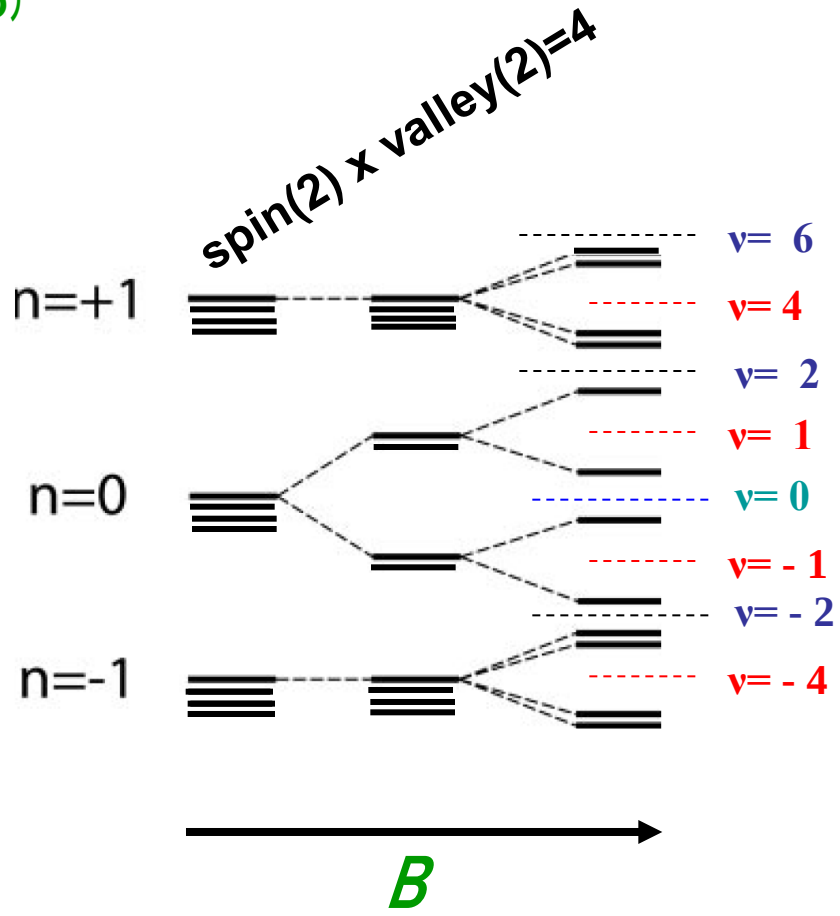
Photo-induced Hall effect ---
rather universal in multi-band systems?

多体効果

Observed splitting of Landau levels

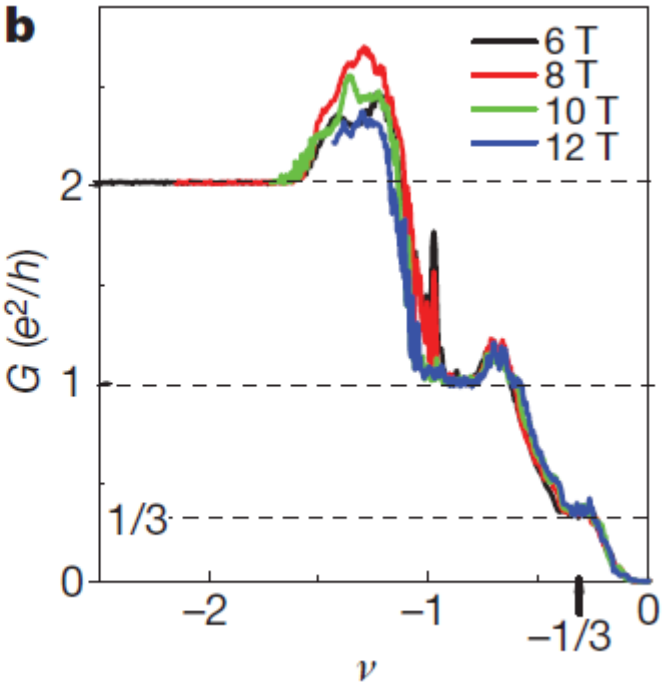
(Zhang et al, PRL 2006)

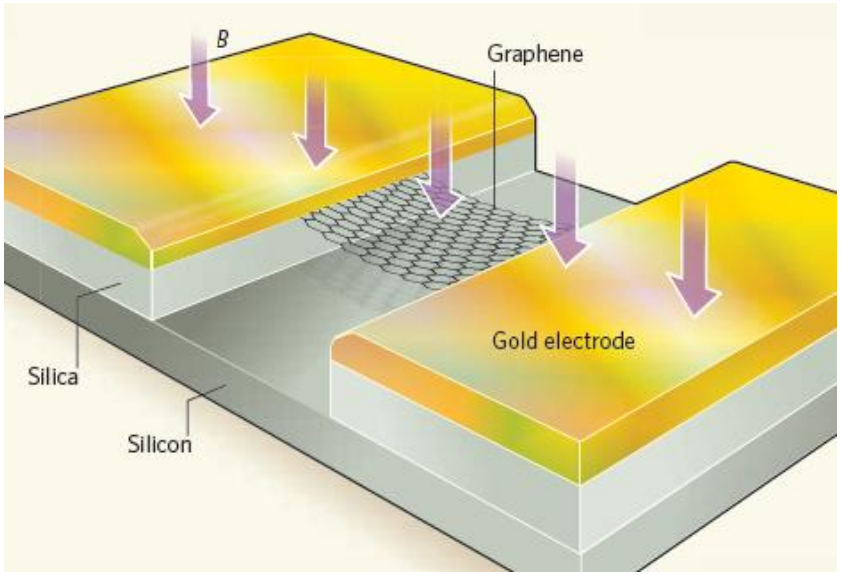




(2) FQHE in graphene

(Suspended graphene: Du et al; Bolotin et al, Nature 2009)



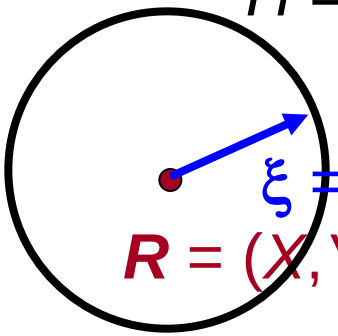


FQHE system ← large quantum zero point

Liquid He ← $[x, p] = i\hbar$

FQHE ← $[X, Y] = -i\hbar$ Non-commutative space !

$H = (1/2m)\pi^2, \quad \pi = \boldsymbol{p} + e\boldsymbol{A}$



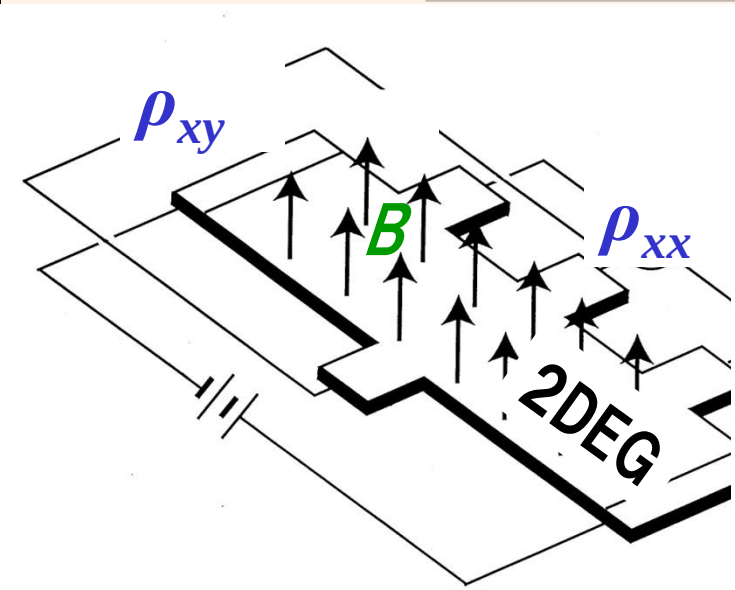
$\xi = -(1/eB)\, \boldsymbol{e}_z \times \Pi, \quad [\xi_x, \xi_y] = -i\ell^2$

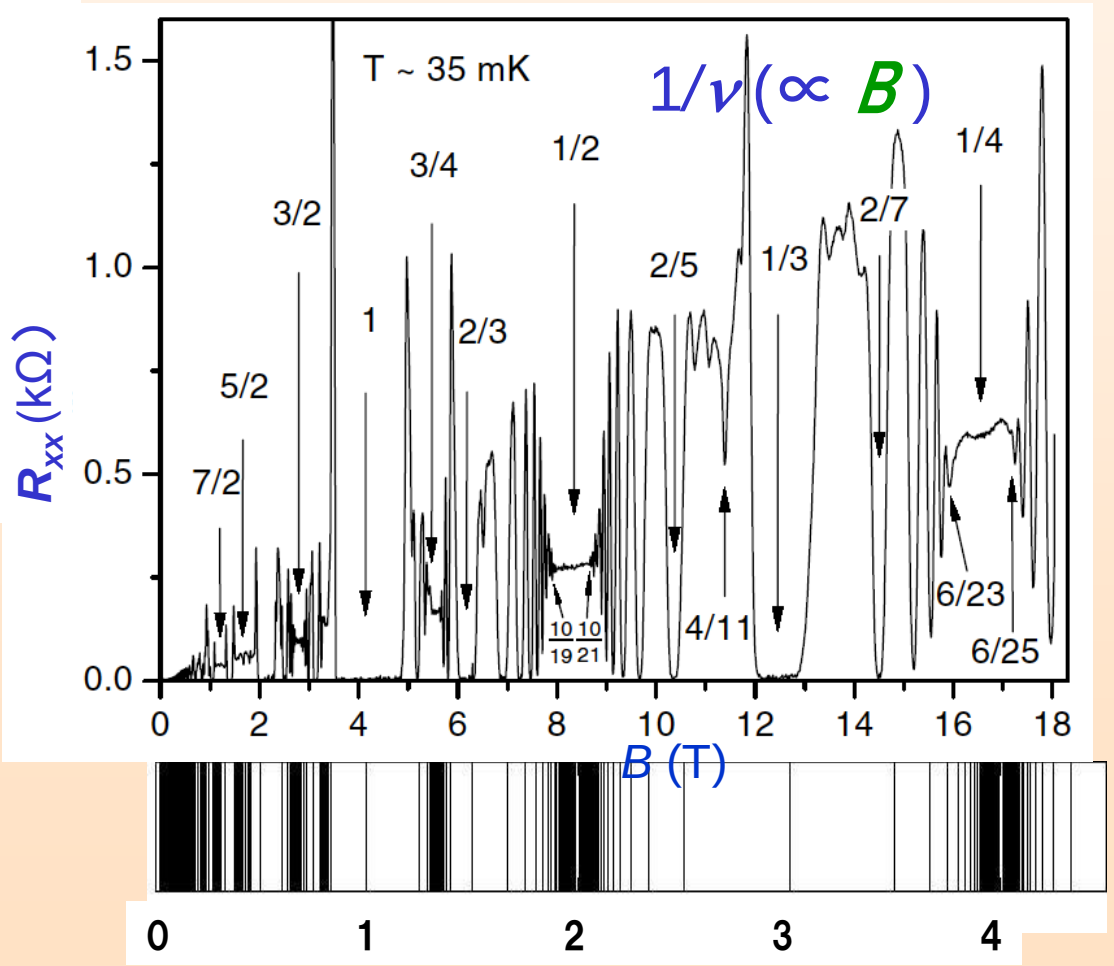
$\boldsymbol{R} = (X, Y), \quad [X, Y] = i\ell^2$

$\ell = (h/eB)^{1/2}$: magnetic length ($\cong$ 80 A for B =10 T)

FQHE = repulsively interacting fermions with uncertain (x, y)

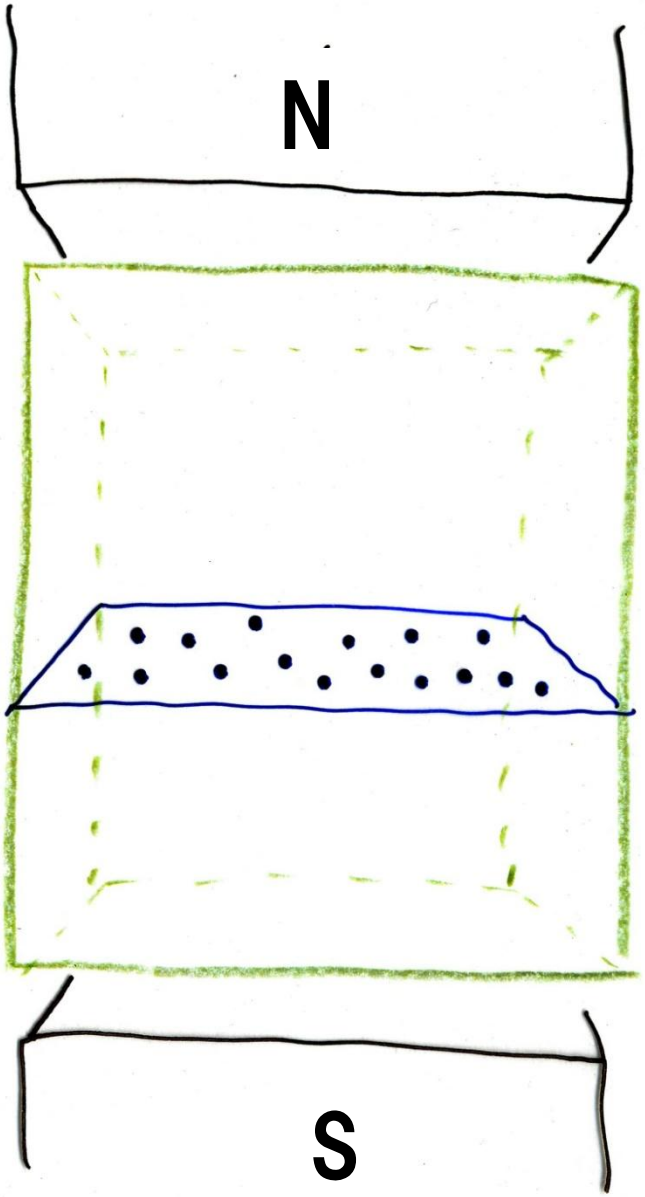
Fractional quantum Hall effect



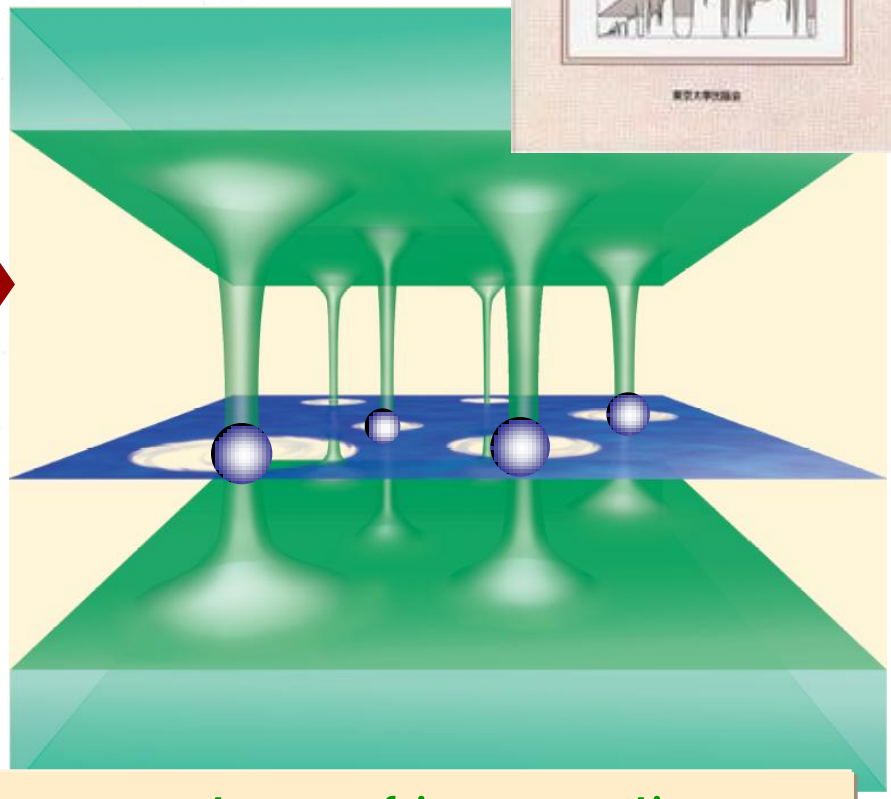


青木 秀夫: グラフェンの物理

Composite fermion picture – flux attachment







A very neat way of incorporating most (short-range) part of int'action for the ``$U/W = \infty$'' system

青木 秀夫: グラフェンの物理

Many-body Hamiltonian for graphene

Coulomb interaction $\mathcal{H}_S = \frac{1}{2}\sum_{\vec{x},\vec{x}'} V(\vec{x}-\vec{x}')n_x n_{x'},$

Spin- and valley-symmetric part $\mathcal{H}_S = \frac{1}{2}\int d^2\vec{x}d^2\vec{x}'\rho_{\text{total}}(\vec{x})V(\vec{x}-\vec{x}')\rho_{\text{total}}(\vec{x}'),$

Valley-symmetry breaking parts $\mathcal{H}_{BS,1} = -u_2\left[J_+^\dagger J_+ + J_-^\dagger J_-\right],$

$\mathcal{H}_{BS,2} = -\int d^2\vec{x}\sum_{\vec{r}} v_1(\vec{r})\rho_{\text{stag}}(\vec{x}+\vec{r})\rho_{\text{stag}}(\vec{x}).$

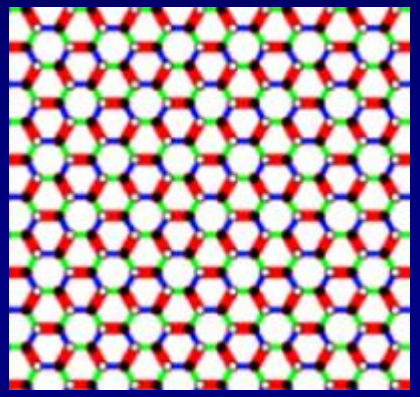
Fractional quantum Hall effect

Haldane's pseudopotential $V_m^{(n)} = \int_0^\infty \frac{dq}{2\pi} qV(q)[F_n(q)]^2 L_m(q^2)e^{-q^2},$

(Abergel et al, Adv Phys 2010)

Mechanisms for split Landau levels in graphene

* Excitonic gap (Gusynin et al, 2006)
* FQHE (Apalkov & Chakraborty, 2006)
* SU(4)-breaking (Nomura & MacDonald, 2007)
* Peierls distortion (Fuchs & Lederer, 2007)
* Bond order (Hatsugai et al, 2007)



$B = 0$, Coulomb interaction

* Exciton condensate (~ chiral condensate in QCD)

(1/N + RG: Son 2007;
RG: Herbut 2009;
Schwinger-Dyson: Khveshchenko 2009;
Monte Carlo: Hands 2008, Drut 2009;
Lattice gauge: Araki & Hatsuda 2010; Araki arXiv:1105.0369)

Future problems

- ✔ Extension to wider topological systems
- ✔ Experimental observations